Math 374: Intro to Representation Theory
MWF 10:00am - 10:50am, 453 HBH. See Syllabus for details
Instructor: Dr. Chelsea Walton. notlaw@rice.edu

[last updated: January 3, 2022]

Expository Paper Information

No prior experience with LaTex is assumed. It takes practice to get comfortable with using LaTex, and you will gain a valuable professional skill after this is achieved.
Getting frustrated at the beginning is typical— Practice is key!
You can do this.

How to use LaTex

There are two ways to create a LaTex document:

(1) Download an editor such as TexShop, or
(2) Use an online editor such as Overleaf.

Overleaf has the advantage that it's easier to get started and to easier to collaborate with others (the latter is not applicable here). But having an offline editor is useful as the "compiling" process is faster and more accurate, and you can work offline.

Here are **training modules** available from U. Waterloo's Faculty of Mathematics:
https://uwaterloo.ca/math-faculty-computing-facility/resources/latex
Modules 1, 2, 8, 11 are necessary to get started, and you can check out the other modules as needed.

Alternatively, **Overleaf has a Quickstart guide** available here:
https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes

There are also numerous **videos on YouTube** on how to get started, including:
LaTex Tutorial for Beginners Full Course: https://youtu.be/fCzF5gDy60g (90 minutes)

A **template of sample article**, "Writing a Math Phase Two Paper" by S. L. Kleiman (MIT), is available here:

For the Tex file: https://math.mit.edu/~notlaw/piiUJM2.tex
The compiled pdf: https://math.mit.edu/~notlaw/piiUJM2.pdf

This resource also contains **valuable tips on how to write a mathematical paper**. (Section 1 is irrelevant for us.)

Expectations for the Paper

Formatting:

- *Margins:* Standard "essay" 1" margins are allowed, so are the standard "article" margins in the template above.
- *Font:* 11pt or 12pt size font.
- *Line Spacing:* Between 1 and 1.25 line spacing; 1.1 - 1.2 is easier on the eyes.

Outline (worth 10% of course grade):

Due Friday, March 11, 2022 at 9:30am sharp via Gradescope

- This should be composed with LaTex.
- Length: **Between 1/2 page and 2 pages.**
- Full sentences are not necessary.
- This should include, at minimum:
  - preliminary title,
  - topic,
  - preliminary section titles,
  - preliminary list of references, and
  - brief description of the material to be discussed in each section.

Draft (worth 15% of course grade):

Due Friday, April 1, 2022 at 9:30am sharp via Gradescope

- This should be composed with LaTex, building on the outline above.
- Length: **At least 3.5 pages, including references.**
- This should include, at minimum:
  - preliminary title,
  - topic,
  - preliminary section titles,
  - references properly formatted in a reference section,
  - at least one reference properly cited,
  - full drafts of some sections of the paper complete, and
  - brief description of the material to be discussed in other sections.

Full Version (worth 25% of course grade):

Due Friday, April 22, 2022 at 9:30am sharp via Gradescope

- This should be composed with LaTex, building on the outline above.
- Length: **7-10 pages, including references.**

Feedback from the instructor will be provided shortly after each stage.

Writing Resources

Many **great tips** are provided in the article "Writing a Math Phase Two Paper" above (Section 1 of that article is irrelevant for us).

Another valuable resource is the **Rice Program in Writing and Communication**. You can book an appointment here: https://pwc.rice.edu/whom-we-serve/undergraduate-students.

Potential Paper Topics and References

Based on the material covered in the first half of the course (through Friday March 4, 2022), some potential topics and references for the expository paper are given below. One should seek out additional references if needed. Of course, you are welcomed to cover a topic (that is related to Representation Theory) not listed here. Please let the instructor know if you need assistance with picking a topic and with choosing references.

A. **Applications of Representation Theory presented in James and Liebeck's text**, such as

1. Application to Group Theory (Chapters 30 and 31), or
2. Application to Molecular Vibration (Chapter 32)

B. **D-modules.** This is the study of representations of "rings of differential operators"; the popular of such rings are the "Weyl algebras". Historically, representations of the Weyl algebras launched the birth of quantum mechanics in the 1920s; more information can be found in Section 3 of the article here: https://arxiv.org/pdf/1808.03172.pdf. See also the first half of the talk available here: https://youtu.be/G2ZX0ZqOBxM. Various paper topics on Weyl algebras, rings of differential operators, and their representations are in Coutinho's text available here, https://bit.ly/3pQ0tbl, including (selections of) Weyl Algebras and their Modules (Chapters 1-5).

C. **Quivers and their representations, and Gabriel's theorem.** This is on the representation theory of directed graphs, i.e. "quivers". A powerful theorem of Gabriel states a quiver has finitely many indecomposable representations if and only if the quiver is of "type ADE". (ADE graphs have a surprising number of connections to various areas of mathematics!) You may only want to concentrate terminology, examples, and the backwards direction of Gabriel's theorem. Potential references include:

- Chapter 6 of P. Etingof et al's text available here: https://klein.mit.edu/~etingof/repb.pdf
- Sections 1.1 and other parts of M. Brion's manuscript available here: https://cel.archives-ouvertes.fr/cel-00440026/document.

D. **Categories of Representations and Adjunction.** As Eugenia Cheng says, "Category Theory is the Mathematics of Mathematics." Representation Theory has a beautiful avatar in Category Theory, and your exposition could begin with basic definitions in Category Theory. Examples are "categories" of modules over a particular group. In this setting, restriction and induction are realized as "functors" (a way of going from one category to another). Moreover, Frobenius Reciprocity is realized as an "adjunction" of functors. Potential references include:

- Chapter 7 of P. Etingof et al's text available here: https://klein.mit.edu/~etingof/repb.pdf
- Sections 1.1 - 1.4 and 4.1 of E. Riehl's text available here: https://emilyriehl.github.io/files/context.pdf