Math 102 Review I

Overview: The first in-class exam will cover Sect. 6.8, 6.9, 7.2–7.6 and 7.8 (only the integral of $\int_0^1 \frac{1}{x^2} \, dx$). But we don’t have so serious request on Chapter 6. The following knowledge points are required for this course, or at least for the coming exam, the related formulas are listed correspondingly.

1. a) The range of arcsin $x$ is $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, that of arctan $x$ is $-\frac{\pi}{2} < y < \frac{\pi}{2}$, and that of arcsec $x$ is $0 \leq y \leq \pi, y \neq \frac{\pi}{2}$.

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}, \quad \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$$
$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}, \quad \int \frac{1}{1+x^2} \, dx = \arctan x + C$$
$$\frac{d}{dx} \text{arcsec } x = \frac{1}{|x|\sqrt{x^2-1}}, \quad \int \frac{1}{x\sqrt{x^2-1}} \, dx = \text{arcsec } |x| + C$$

2. The derivative of hyperbolic functions:

$$\frac{d}{dx} \sinh x = \cosh x \quad \int \cosh x \, dx = \sinh x + C$$
$$\frac{d}{dx} \tanh x = \sec h^2 x \quad \int \sec h^2 x \, dx = \tanh x + C$$
$$\frac{d}{dx} \text{sech } x = - \sec x \tanh x \quad \int \sec h x \tanh x \, dx = - \sec h x + C$$

3. Integration by substitution

Generally, the original problem is: $\int f(g(x))g'(x) \, dx$

Let $u = g(x) \quad du = g'(x) \, dx$

(don’t forget substitute back to $x$)

4. Integration by parts.

Get familiar with those basic types of problems. Pay attention to those with $\sin x, \cos x, e^x, \ln x, \sinh x, \cosh x$ involved.

And $\int \sec x \, dx = \ln |\sec x + \tan x| + C = \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C$

5. The classification of $\int \sin^n x \cos^m x \, dx$ and $\int \tan^n x \sec^m x \, dx$.

For $\int \sin^n x \cos^m x \, dx$, we require you to get familiar with the case that one of $n$ and $m$ is odd.

For $\int \tan^n x \sec^m x \, dx$, we require you to get familiar with the case that $m$ is even.


Generally, we require you to know how to integrate the rational function with the denominator in linear factors or at most $x^2 + a$.

7. Trigonometric Substitution

<table>
<thead>
<tr>
<th>Involved term</th>
<th>Substitution</th>
<th>Identity formula</th>
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</thead>
<tbody>
<tr>
<td>$\sqrt{a^2 - u^2}$</td>
<td>$u = a \sin \theta$</td>
<td>$\sqrt{a^2 - u^2} = \sqrt{a^2 - a^2 \sin^2 \theta} = a \cos \theta$</td>
</tr>
<tr>
<td>$\sqrt{a^2 + u^2}$</td>
<td>$u = a \cos \theta$</td>
<td>$\sqrt{a^2 + u^2} = \sqrt{a^2 + a^2 \tan^2 \theta} = a \sec \theta$</td>
</tr>
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<td>$u = a \sec \theta$</td>
<td>$\sqrt{u^2 - a^2} = \sqrt{a^2 \sec^2 \theta - a^2} = \pm a \tan \theta$</td>
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Note: 1. the reference triangle is very helpful (for pictures, please see the figures in Page 518 to Page 520 of the textbook, we use figure 7.6.3 with a = 5 to explain a case).

Textbook, Page 519, figure 7.6.3: if a = 5, let u = 5 tan θ, we know that tan θ = \frac{u}{5}, but sometimes we need to know what sin θ is to re-substitute back to u; it is easily seen from the figure that sin θ = \frac{u}{\sqrt{25+u^2}}, also cos θ = \frac{5}{\sqrt{25+u^2}}.

2. √(u^2 - a^2) = √(a^2 sec^2 θ - a^2) = ±a tan θ dont worry about the sign, you may always take it as positive in indefinite integral.

8. \int_0^1 \frac{1}{x^\alpha} dx = \begin{cases} +\infty & \alpha \geq 1 \\ \frac{1}{1-\alpha} & \alpha < 1 \end{cases}

Appendix: Some fundamental formulas:

\sin^2 θ + \cos^2 θ = 1 \quad \Rightarrow \quad \cos θ = \sqrt{1 - \sin^2 θ} \quad (-\frac{\pi}{2} \leq θ \leq \frac{\pi}{2})
\tan^2 θ + 1 = \sec^2 θ \quad \Rightarrow \quad \sec θ = \sqrt{\tan^2 θ + 1} \quad (-\frac{\pi}{2} < θ < \frac{\pi}{2})
\cos 2θ = 1 - 2\sin^2 θ = 2\cos^2 θ - 1 \quad \Rightarrow \quad \sin^2 θ = \frac{1 - \cos 2θ}{2} \quad \text{and} \quad \cos^2 θ = \frac{1 + \cos 2θ}{2}

Focus on this Review and the notes examples, dont worry about the homework problems.