§6.3

#8. Calculate the mass of the object bounded by the cylinder \( x^2 + y^2 = 2x \) and the cone \( x^2 + y^2 = z^2 \) if the density is given by \( \delta(x, y, z) = \sqrt{x^2 + y^2} \).

Call the region \( W \). In \( x, y, z \) coordinates, this region is given by

\[
0 \leq x \leq 2 \\
-\sqrt{2x - x^2} \leq y \leq \sqrt{2x - x^2} \\
-\sqrt{x^2 + y^2} \leq z \leq \sqrt{x^2 + y^2}.
\]

Switching to polar coordinates \( x = r \cos \theta, y = r \sin \theta, z = z \), this same region is given by:

\[
-\pi/2 \leq \theta \leq \pi/2 \\
0 \leq r \leq 2 \cos \theta \\
-r \leq z \leq r.
\]

Call the region in polar coordinates \( W^* \). Note that the reason \(-\pi/2 \leq \theta \leq \pi/2\) is that the equation \( r = 2 \cos \theta \) traces out the unit disk centered at \((1, 0)\) with period \( \pi \). So we know that we want \( \theta \) to range over an interval of length \( \pi \). I choose \(-\pi/2 \leq \theta \leq \pi/2\) so that \( r \) would always be non-negative. Using this change of coordinates we calculate:

\[
\int \int \int_W \sqrt{x^2 + y^2} \, dz \, dy \, dx = \int \int \int_{W^*} r^2 \, dz \, dr \, d\theta = 3\pi.
\]