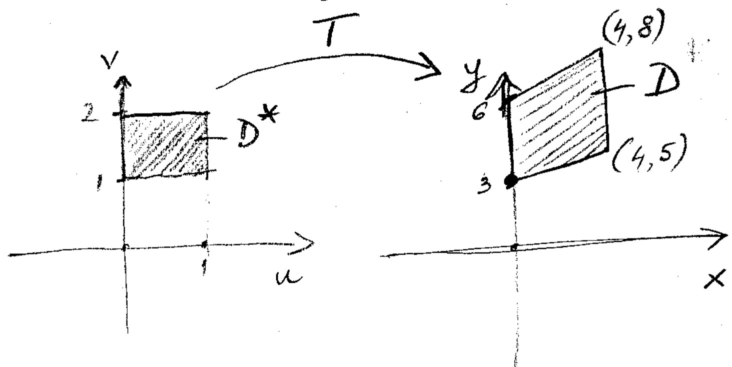
Math 212 - Hw #9

$\boxed{1/6.2.}$

$$\iint_D e^{x^2+y^2}\,dx\,dy = \int_{r=0}^{1}\int_{\theta=0}^{2\pi} e^{r^2} r\,d\theta\,dr =$$

$$= 2\pi \int_0^1 e^{r^2} r\,dr = 2\pi \cdot \frac{e^{r^2}}{2}\Big|_0^1 = \boxed{\pi(e-1).}$$

$\boxed{3/6.2.}$



$T(0,1) = (0,3)$

$T(0,2) = (0,6)$

$T(1,1) = (4,5)$

$T(1,2) = (4,8)$

Notice that $D:\quad 0 \le x \le 4$

$$\frac{1}{2}x+3 \le y \le \frac{1}{2}x+6$$

(a)

$$\iint_D xy\,dx\,dy = \iint_{D^*} x(u,v)\,y(u,v)\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

$$= \int_0^1\int_1^2 4u\,(2u+3v)\cdot 12\,dv\,du$$

$$= 48\int_0^1\int_1^2 2u^2+3uv\,dv\,du$$

$$= 48\int_0^1 2vu^2 + 3u\cdot\frac{v^2}{2}\Big|_1^2\,du$$

$$= 48\int_0^1 4u^2+6u-2u^2-\frac{3u}{2}\,du$$

$$= 48\int_0^1 2u^2+\frac{9}{2}u\,du = 48\cdot\left(\frac{2u^3}{3}+\frac{9}{4}u^2\right)\Big|_0^1 =$$

$$= 48\left(\frac{2}{3}+\frac{9}{4}\right) = 48\cdot\frac{35}{12} = \boxed{140}$$

$\boxed{9/6.2.}$ $$\iint_D (x^2+y^2)^{3/2}\,dx\,dy \overset{\text{polar}}{\underset{\text{coord.}}{=}} \int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r^2)^{3/2}\, r\,d\theta\,dr$$

$$= \int_0^2\int_0^{2\pi} r^4\,d\theta\,dr = 2\pi\int_0^2 r^4\,dr = 2\pi\cdot\frac{32}{5} = \boxed{\frac{64\pi}{5}}.$$

$\boxed{13/6.2.}$ Use cylindrical coordinates:

$$\iiint_D z\cdot e^{x^2+y^2}\,dx\,dy\,dz = \int_{r=0}^{2}\int_{\theta=0}^{2\pi}\int_{z=2}^{3} z\cdot e^{r^2}\cdot r\,dz\,d\theta\,dr$$

$$= \int_0^2\int_0^{2\pi} \frac{z^2}{2}e^{r^2} r\Big|_2^3\,d\theta\,dr = 2\pi\int_0^2 \frac{5}{2}e^{r^2} r\,dr$$

$$= 5\pi\cdot\frac{e^{r^2}}{2}\Big|_0^2 = \boxed{\frac{5\pi}{2}(e^4-1)}.$$

$\boxed{14/6.2}$ Use polar coordinates to get

$$\int_{r=0}^{1}\int_{\theta=0}^{2\pi}(1+r^2)^{3/2}\, r\,d\theta\,dr = 2\pi\int_0^1 r(1+r^2)^{3/2}\,dr$$

$$= 2\pi\cdot\frac{(1+r^2)^{5/2}}{5}\Big|_0^1 = \boxed{\frac{2\pi}{5}(4\sqrt{2}-1)}.$$

$\boxed{19/6.2}$ Use cylindrical coordinates to get:

$$\int_{r=0}^{\sqrt{2}}\int_{\theta=0}^{2\pi}\int_{z=-2}^{3}(r^2+z^2)\,r\,dz\,d\theta\,dr =$$

$$= \int_0^{\sqrt{2}}\int_0^{2\pi} r^3 z + r\cdot\frac{z^3}{3}\Big|_{-2}^{3}\,d\theta\,dr = 2\pi\int_0^{\sqrt{2}} 5r^3 + \frac{35}{3}r\,dr$$

$$= 2\pi\cdot\left(\frac{5r^4}{4}+\frac{35}{3}\cdot\frac{r^2}{2}\right)_0^{\sqrt{2}} = 2\pi\left(\frac{50}{3}\right) = \boxed{\frac{100\pi}{3}}$$

21/6.2 Use spherical coordinates to get:

$$\int_{\rho=0}^{1}\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi} \frac{1}{\sqrt{2+\rho^2}}\,\rho^2\sin\varphi\, d\varphi\, d\theta\, d\rho$$

$$= 2\pi\int_0^1 \frac{2\rho^2}{\sqrt{2+\rho^2}}\,d\rho \;\underset{58}{\overset{\text{formula}}{=\!=}}\; 4\pi\left(\frac{\rho\sqrt{\rho^2+2}}{2} - \log\left(\rho+\sqrt{\rho^2+2}\right)\right)\Bigg|_0^1$$

$$= \boxed{4\pi\left(\frac{\sqrt{3}}{2} - \log\left(1+\sqrt{3}\right) + \log\left(\sqrt{2}\right)\right)}.$$

23/6.2 Use spherical coordinates to get:

$$\int_{\rho=b}^{a}\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi} \frac{\rho^2\sin\varphi\, d\varphi\, d\theta\, d\rho}{(\rho^2)^{3/2}} =$$

$$= 2\pi\int_b^a\int_0^\pi \frac{1}{\rho}\sin\varphi\, d\varphi\, d\rho = 2\pi\int_b^a \frac{2}{\rho}\,d\rho =$$

$$= 4\pi \log\rho\Big|_b^a = 4\pi(\log a - \log b) = \boxed{4\pi\log\frac{a}{b}}.$$

25/6.2

$$\int_{\rho=b}^{a}\int_{0}^{2\pi}\int_{\varphi=0}^{\pi} \rho\cdot e^{-\rho^2}\cdot\rho^2\sin\varphi\, d\varphi\, d\theta\, d\rho$$

$$= 2\pi\int_b^a 2\rho^3 e^{-\rho^2}\,d\rho = 2\pi\int_{b^2}^{a^2} u\cdot e^{-u}\,du$$

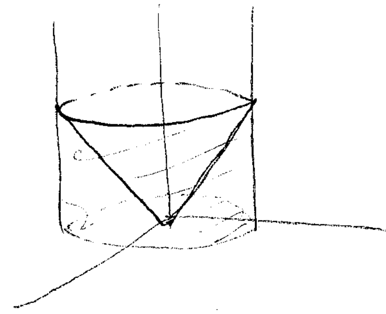
(let $u=\rho^2$; $2\rho\, d\rho = du$)

$$= 2\pi\left(-u\cdot e^{-u}\Big|^{1} + \int_{b^2}^{a^2} e^{-u}\,du\right) =$$

$$= 2\pi\left(b^2 e^{-b^2} - a^2 e^{-a^2} + e^{-b^2} - e^{-a^2}\right).$$

$$= \boxed{2\pi\left[(b^2+1)e^{-b^2} - (a^2+1)e^{-a^2}\right]}.$$

26/6.2 (a) $\iiint_B z\,dx\,dy\,dz =$

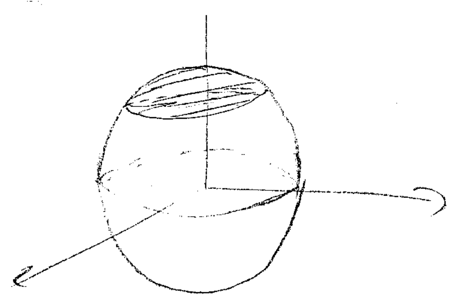


$$= \int_{r=0}^{1}\int_{\theta=0}^{2\pi}\int_{z=0}^{r} z\,r\,dz\,d\theta\,dr$$

$$= 2\pi\int_0^1 \frac{r^3}{2}\,dr = \pi\cdot\frac{r^4}{4}\Big|_0^1$$

$$= \boxed{\frac{\pi}{4}}$$

(b) $x = r\cos\theta$, $y = r\sin\theta$, $z = z$



$$\iiint_W (x^2+y^2+z^2)^{-1/2}\,dx\,dy\,dz =$$

$$= \int_{z=1/2}^{1}\int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{1-z^2}} \frac{r}{\sqrt{r^2+z^2}}\,dr\,d\theta\,dz$$

$$= 2\pi\int_{1/2}^{1}\int_0^{\sqrt{1-z^2}} \frac{r}{\sqrt{r^2+z^2}}\,dr\,dz = 2\pi\int_{1/2}^{1} \frac{\sqrt{r^2+z^2}}{2}\Big|_0^{\sqrt{1-z^2}}\,dr$$

$$= \pi\int_{1/2}^{1}(1-z)\,dz = \boxed{\frac{\pi}{4}}$$