Math 212 - Hw # 2

$\boxed{3/1.3.}$ $\bar{a} = \bar{i} - 2\bar{j} + \bar{k}$; $\bar{b} = 2\bar{i} + \bar{j} + \bar{k}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \underline{-3\bar{i} + \bar{j} + 5\bar{k}}$$

$\boxed{7/1.3.}$ Vol = absolute value of $\begin{vmatrix} 2 & 1 & -1 \\ 5 & 0 & -3 \\ 1 & -2 & 1 \end{vmatrix} = |-10| = \underline{\underline{10.}}$

$\boxed{15b/1.3.}$ The plane is perpendicular to $\bar{i} + 2\bar{j} + 3\bar{k}$ and passes through $(1,1,1)$, hence the equation is:

$$1 \cdot (x-1) + 2(y-1) + 3(z-1) = 0, \text{ or}$$

$$\underline{x + 2y + 3z - 6 = 0.}$$

$\boxed{16b/1.3.}$ A normal vector to the plane is:

$$\bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & -1 & -2 \\ 3 & -2 & 1 \end{vmatrix} = -5\bar{i} - 5\bar{j} + 5\bar{k}$$

Hence the equation is:

$$-5(x-1) - 5(y-2) + 5(z-0) = 0, \text{ or}$$

$$\underline{-5x - 5y + 5z + 15 = 0}, \text{ or}$$

$$\underline{x + y - z - 3 = 0.}$$

$\boxed{20/1.3.}$ We need to solve the system:

$$3x + 2y + z = 2$$
$$x + 4y - z = 2$$

Students knowing the row-reduction algorithm can solve the system that way. Otherwise, add the two equations to get

$$4x + 6y = 4$$

Let $y = t$ (free parameter) $\Rightarrow x = 1 - \frac{3}{2}t$

Then $z = x + 4y - 2 = 1 - \frac{3}{2}t + 4t - 2 = \frac{5}{2}t - 1$

The parametric equation of the line of intersection is:

$$x = 1 - \frac{3}{2}t$$
$$y = t$$
$$z = -1 + \frac{5}{2}t$$

<u>Remark</u>: The answer depends on the way we choose the parameter t, hence the equation is not unique, but it gives a unique line.

$\boxed{25/1.3.}$ A normal to the plane is given by the direction of the line, hence $\bar{n} = \bar{i} - 2\bar{j} + 3\bar{k}$.

Plane equation: $1\cdot(x-1) - 2(y-2) + 3(z+3) = 0$, or

$$\underline{x - 2y + 3z + 12 = 0.}$$

$\boxed{26/1.3}$ Since the normal to the plane is given by $\bar{n} = (3,-1,-2)$, the line has direction given by $\bar{n}$. Its equation is:

$$x = 1 + 3t$$
$$y = -2 - t \qquad \text{or} \qquad \bar{l}(t) = (1,-2,-3) + t(3,-1,-2)$$
$$z = -3 - 2t$$

$\boxed{27/1.3}$ Pick three points in the plane (from those lines):
For example $(0,1,-2)$, $(2,-1,0)$ and $(2,4,-3)$

A normal to the plane: $\bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = -4\bar{i} + 6\bar{j} + 10\bar{k}$

Plane equation: $-4(x-0) + 6(y-1) + 10(z+2) = 0$, or

$$\underline{-4x + 6y + 10z + 14 = 0.}$$

$\boxed{28/1.3}$ $\text{distance} = \dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}} = \dfrac{|2-2-2+5|}{\sqrt{1+4+4}} = \dfrac{3}{3} = \underline{1}$

$\boxed{29/1.3}$ Notice that a normal to the plane is perpendicular to $(3,2,4)$ (the direction of the line) and to $(2,1,-3)$ (the normal to the other plane). Hence

$\bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = -10\bar{i} + 17\bar{j} - \bar{k}$ and the equation is:

$$-10(x+1) + 17(y-1) - (z-2) = 0 \quad \text{or}$$

$$\underline{-10x + 17y - z - 25 = 0.}$$