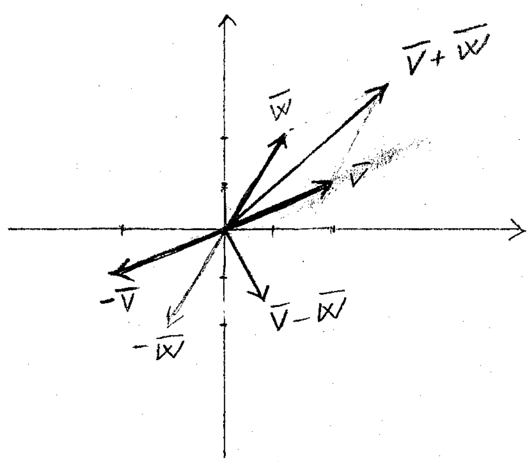
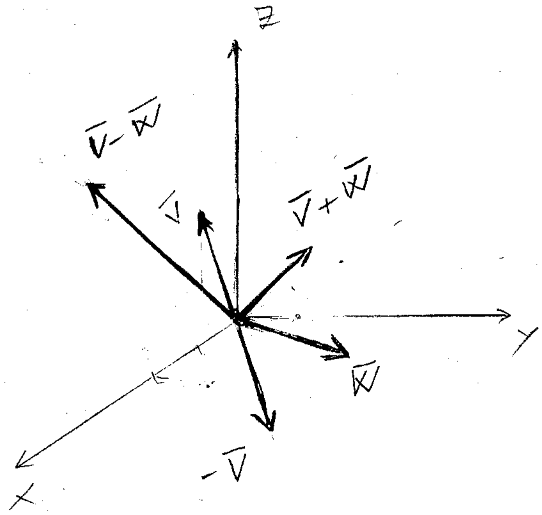
Math 212 - Hw #1

$\boxed{5/1.1}$



$\boxed{8/1.1.}$



$\boxed{15/1.1}$ The parametric equations are:

$$x = -1 + (1+1)t = -1+2t$$
$$y = -1 + (-1+1)t = -1$$
$$z = -1 + (2+1)t = -1+3t$$

or, in vector notation: $\bar{\ell}(t) = (2t-1)\bar{i} - \bar{j} + (3t-1)\bar{k}$

$\boxed{21/1.1.}$ We look for s, t such that

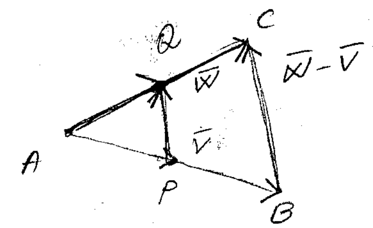
$$3t+2 = 3s-1; \quad t-1 = s-2; \quad 6t+1 = s$$

The second equation gives us $s = t+1$, and substituting into the third: $6t+1 = t+1 \Rightarrow t=0$.

So $t=0$ and $s=1$ is a valid solution (they also satisfy the first equation).

$\therefore$ The lines intersect at the point $\underline{(2,-1,1)}$

26/1.1 Consider the triangle ABC with the midpoint of AB denoted by P and the midpoint of AC denoted by Q. Let $\overrightarrow{AB} = \bar{v}$, $\overrightarrow{AC} = \bar{w}$ (as free vectors)



Then $\overrightarrow{AP} = \frac{1}{2}\bar{v}$, $\overrightarrow{AQ} = \frac{1}{2}\bar{w}$, $\overrightarrow{PQ} = \frac{1}{2}\bar{w} - \frac{1}{2}\bar{v}$

Since $\overrightarrow{BC} = \bar{w} - \bar{v}$, notice that $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{BC}$, hence the segment PQ is parallel to BC and $\|\overrightarrow{PQ}\| = \frac{1}{2}\|\overrightarrow{BC}\|$

3/1.2

$$\cos\theta = \frac{(7\bar{j}+19\bar{k})\cdot(-2\bar{i}-\bar{j})}{\|7\bar{j}+19\bar{k}\|\,\|-2\bar{i}-\bar{j}\|} = \frac{-7}{\sqrt{410}\cdot\sqrt{5}} = \frac{-7}{\sqrt{2050}}$$

$$\theta = \cos^{-1}(-7/\sqrt{2050}) \approx 99^\circ$$

9/1.2

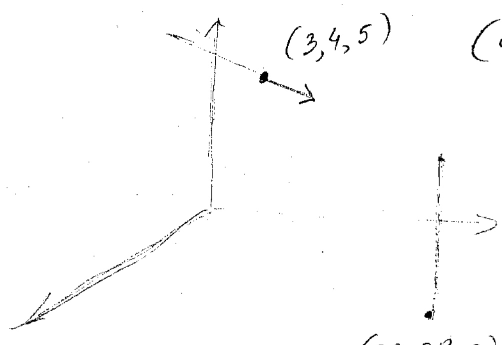
$$\|\bar{u}\| = \sqrt{1+9+1} = \underline{\sqrt{11}}, \quad \|\bar{v}\| = \sqrt{4+9+49} = \underline{\sqrt{62}}$$

$$\bar{u}\cdot\bar{v} = (-\bar{i}+3\bar{j}+\bar{k})\cdot(-2\bar{i}-3\bar{j}-7\bar{k}) = 2-9-7 = \underline{-14}$$

15/1.2

$$\operatorname{proj}_{\bar{u}}\bar{v} = \frac{\bar{v}\cdot\bar{u}}{\|\bar{u}\|^2}\,\bar{u} = \frac{-4}{3}\,\bar{u} = \boxed{\frac{4}{3}\bar{i} - \frac{4}{3}\bar{j} - \frac{4}{3}\bar{k}}$$

21/1.2



(a) The position of the airplane after t hours is:

$$(3+400t,\ 4+500t,\ 5-t)$$

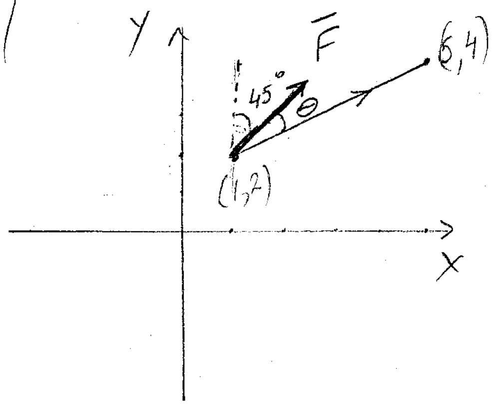
We need t such that:

$$3+400t = 23;\quad 4+500t = 29$$

$\boxed{\text{Hence } t = \frac{1}{20} = 3\,min;\ \text{so } 12{:}03\,pm}$

(b) The position for $t = \frac{1}{20}$ is $(23, 29, 4.95)$; $\boxed{z = 4.95\ Km}$

27/1.2



(a) $\bar{F} = F_1 \bar{i} + F_2 \bar{j}$

$F_1 = \|\bar{F}\| \cos \frac{\pi}{4} = 6 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$

$F_2 = \|\bar{F}\| \sin \frac{\pi}{4} = 6 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$

Hence $\bar{F} = 3\sqrt{2}\,\bar{i} + 3\sqrt{2}\,\bar{j}$

(b) $$\cos\theta = \frac{\bar{F} \cdot \bar{D}}{\|\bar{F}\| \cdot \|\bar{D}\|} = \frac{(3\sqrt{2}\,\bar{i} + 3\sqrt{2}\,\bar{j}) \cdot (4\bar{i} + 2\bar{j})}{6 \cdot \sqrt{16+4}} =$$

$$= \frac{12\sqrt{2} + 6\sqrt{2}}{6\sqrt{20}} = \frac{18\sqrt{2}}{6\sqrt{20}} = \frac{3}{\sqrt{10}}$$

Hence $\theta = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right) \approx 18.4°$

(c) Already done in (b). $\bar{F} \cdot \bar{D} = 18\sqrt{2}$

$\|\bar{F}\| \cdot \|\bar{D}\| \cos\theta = 18\sqrt{2}$