1. If u is harmonic on a connected open set, and if u^2 is also harmonic, then prove that u is constant.

2. Prove that if u is harmonic on a connected open set Ω and if u vanishes on some small disc, then u is identically zero on all of Ω.

3. If u is harmonic and bounded on all of $\mathbb{C}$, then u is identically constant. (Liouville's theorem for harmonic functions.)

4. Compute a Poisson integral formula for $\Omega = \{z : |z| < 1, \mathrm{Im} z > 0\}$ by mapping Ω conformally to the disc.

5. Let u be a continuous function on an open set Ω and harmonic on $\Omega - \{z_0\}$. Prove that u is in fact harmonic on all of Ω.

6. Let u be a positive harmonic function on the unit disc and suppose that $u(0) = \alpha$. How large can $u(3/4)$ be? How small can it be? What is the best possible bound? What function realizes that bound?

7. Let $g(z)$ be analytic in the right half-plane with $|g(z)| < 1$ for all such z. If $g(1) = 0$, how large can $g(2)$ be?