

Solutions to $x^3 + y^3 + z^3 = 1$ in terms of s and t:

$$x = \frac{t - \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right)^2}{t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}$$

$$y = \frac{s + t + \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right)^2}{t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}$$

$$z = \frac{-(s+t) \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}{t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}$$

$$\text{Check: } \left(\frac{t - \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right)^2}{t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}\right)^3 + \left(\frac{s + t + \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right)^2}{t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}\right)^3 + \left(\frac{-(s+t) \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}{t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1}\right)^3 =$$

$$= \frac{\left(t - \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right)^2\right)^3 + \left(s + t + \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right)^2\right)^3 + \left(- (s+t) \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1\right)^3}{\left(t \left(\frac{s^2}{3} + \frac{st}{3} + \frac{t^2}{3}\right) - 1\right)^3}$$

$$= \frac{t^3 - \frac{1}{3}t^6 + \frac{1}{27}t^9 + st^2 + s^2t - \frac{2}{3}st^5 + \frac{1}{9}st^8 - s^2t^4 - \frac{2}{3}s^3t^3 - \frac{1}{3}s^4t^2 + \frac{2}{9}s^2t^7 + \frac{7}{27}s^3t^6 + \frac{2}{9}s^4t^5 + \frac{1}{9}s^5t^4 + \frac{1}{27}s^6t^3 - 1}{t^3 - \frac{1}{3}t^6 + \frac{1}{27}t^9 + st^2 + s^2t - \frac{2}{3}st^5 + \frac{1}{9}st^8 - s^2t^4 - \frac{2}{3}s^3t^3 - \frac{1}{3}s^4t^2 + \frac{2}{9}s^2t^7 + \frac{7}{27}s^3t^6 + \frac{2}{9}s^4t^5 + \frac{1}{9}s^5t^4 + \frac{1}{27}s^6t^3 - 1^3}$$

$$= 1$$