March 19, 2008

1)Let $Y = \mathbb{A}^2(\mathbb{C})$ be the affine plane with coordinates x and y.

a. Compute the blowup $\tilde{Y}_1 \to Y$ along the subscheme with ideal $\langle x^2, y^3 \rangle$.

b. Compute the blowup of $\tilde{Y}_2 \to Y$ along the subscheme with ideal $\langle x^2, xy^2, y^3 \rangle$.

c. Show there is a morphism $\tilde{Y}_2 \to \tilde{Y}_1$ over Y.

2)Let $Y = \{(x, y, z) : xy - z^2 = 0\} \subset \mathbb{A}^3(\mathbb{C})$. Compute the blowup of Y along the subschemes with ideals

a. $\langle x, z \rangle$;

b. $\langle x \rangle$;

c. $\langle x, y \rangle$.

3)Let $Y \simeq \mathbb{A}^n(k)$ be affine space with coordinates $x_1, \ldots, x_n$. Let X be the reduced subscheme $P_1 \cup P_2$ where

$$P_1 = \{x_1 = \ldots = x_i = 0\} \quad P_2 = \{x_{i+1} = \ldots = x_n = 0\}.$$

a. Let $\beta : \tilde{Y} \to Y$ be the blowup along X. Show that $\tilde{Y}$ is smooth and describe the fiber $\beta^{-1}(0)$.

b. Consider the sequence of blowups

$$\beta_\circ : Y_2 \to Y_1 \to Y$$

where Y_1 is the blow-up along P_1 and Y_2 the blow-up of Y_1 along the proper transform of P_2. Compute the fiber of $\beta_\circ^{-1}(0)$.

c. Discuss whether Y_2 and $\tilde{Y}$ are isomorphic over Y.

4)Let $Y = \{(x, y, z) : xy - z^2 = 0\} \subset \mathbb{P}^3(\mathbb{C})$, $X = \{x = z = 0\}$, and $\iota : X \to Y$ the corresponding inclusion. Can you define a Gysin map $\iota^* : A_2(Y) \to A_1(X)$ compatible with the pull-back operation on Cartier divisors? What if you allow rational coefficients, i.e., is there a Gysin map $\iota^* : A_2(Y) \otimes \mathbb{Q} \to A_1(X) \otimes \mathbb{Q}$?

5)Show that any morphism of nonsingular varieties is a local complete intersection morphism.

6)Let $Y = \mathbb{A}^4$, $X = \{x_1 - x_3 = x_2 - x_4 = 0\}$ and $V = \{x_1x_3 = x_1x_4 = x_2x_3 = x_2x_4 = 0\}$. Compute $i(0, X \cdot V; Y)$ *using the definition.*

7)Consider the linear series of cubics on $\mathbb{P}^2(\mathbb{C})$ with basis

$$\left\langle x^2y, y^2z, z^2x, xyz \right\rangle$$

and base locus B. Consider the induced morphism $\phi : \mathrm{Bl}_B(\mathbb{P}^2) \to \mathbb{P}^3$ obtained by resolving the indeterminacy. Compute the degree of the image of ϕ and show that ϕ is a closed imbedding.

8)Let $E = \mathcal{O}_{\mathbb{P}^4}(2)^{\oplus 6}$ and $C \subset \mathbb{P}^4$ the subscheme given by the vanishing of the section $s \in \Gamma(\mathbb{P}^4, E)$ associated to the sextuple

$$(x_0x_2 - x_1^2, x_0x_3 - x_1x_2, x_0x_4 - x_1x_3, x_1x_3 - x_2^2, x_1x_4 - x_2x_3, x_2x_4 - x_3^2).$$

Compute the excess normal bundle of the fiber square

$$\begin{array}{ccc} C & \to & \mathbb{P}^4 \\ \downarrow & & \downarrow s \\ \mathbb{P}^4 & \xrightarrow{0} & E. \end{array}$$

Please keep in mind that $C \simeq \mathbb{P}^1$ so any vector bundle on C is isomorphic to

$$\mathcal{O}_{\mathbb{P}^1}(a_1) \oplus \mathcal{O}_{\mathbb{P}^1}(a_2) \oplus \ldots \oplus \mathcal{O}_{\mathbb{P}^1}(a_r).$$