February 6, 2008

1)Let $E = \mathcal{O}_{\mathbb{P}^1}(a) \oplus \mathcal{O}_{\mathbb{P}^1}(b)$ be a rank-two vector bundle on $\mathbb{P}^1$ with $a \leq b$. Give a concrete imbedding of $\mathbb{P}(E)$ into projective space; write down equations defining the image. *Hint:* It is possibile to realize it as a surface of degree $b-a+2$ in $\mathbb{P}^{b-a+3}$.

2)Let V be a three-dimensional vector space and $\mathbb{F}l(V)$ the space of complete flags. Show this is isomorphic to

$$\{[x_0, x_1, x_2], [p_0, p_1, p_2] : x_0p_0 + x_1p_1 + x_2p_2 = 0\} \subset \mathbb{P}^2 \times \mathbb{P}^2.$$

3)Consider the surface

$$X = \{(w, x, y, z) : w^3 = xyz\} \subset \mathbb{P}^3(\mathbb{C})$$

and the effective Weil-divisors $D_1 = \{w = x = 0\}$ and $D_2 = \{w = y = 0\}$. Compute $(D_1.D_2)_X$ and $(D_1.D_1)_X$ using Mumford's technique.

4)Consider the threefold

$$X = \{[v, w, x, y, z] : wx = yz\} \subset \mathbb{P}^4(\mathbb{C})$$

and the resolution

$$\tilde{X} = \mathrm{Bl}_p(X) \xrightarrow{\beta} X$$

where $p = [1, 0, 0, 0, 0]$. Can you define a pull-back operation

$$\beta^* : A_2(X) \to A_2(\tilde{X})$$

extending the homomorphism

$$\beta^* : \mathrm{Pic}(X) \to \mathrm{Pic}(\tilde{X})?$$