January 23, 2008

1)In class, we showed that the nodal plane cubic

$$X = \{(x, y) : y^2 = x^2 + x^3\} \subset \mathbb{A}^2(\mathbb{C})$$

satisfies $\mathrm{Pic}(X) \simeq \mathbb{G}_m = \mathbb{C}^*$. Exhibit an explicit Cartier divisor corresponding to $-1 \in \mathbb{C}^*$.

2)Consider the 'triangle curve'

$$C = \{xyz = 0\} \subset \mathbb{P}^2(\mathbb{C}).$$

Compute $\mathrm{Pic}(C)$ and $A_0(C)$.

3)Consider the cuspidal plane cubic

$$X = \{(x, y) : y^2 = x^3\}.$$

Show that $\mathrm{Pic}(X) \simeq \mathbb{G}_a = \mathbb{C}$.

4)Consider the surface

$$S = \{(x, y, z) : xy = z^2\}.$$

Show that the natural map

$$\mathrm{Pic}(S) \to A_1(S)$$

is not surjective by showing that the Weil divisor

$$D = \{x = z = 0\}$$

is not Cartier. *Hint:* It suffices to show that D is not defined by a single equation.

5)Let X be a d-dimensional variety that is locally factorial, i.e., each local ring $\mathcal{O}_{x,X}, x \in X$ is a unique factorization domain. Show that

$$\mathrm{Pic}(X) \to A_{d-1}(X)$$

is surjective.