January 16, 2008

1)Let $\mathbb{F}_q$ be a finite field and $V \subset \mathbb{A}^n(\mathbb{F}_q)$ a subvariety of dimension $k < n$. Show that for $d \gg 0$,

$$[V] = 0 \text{ in } A_k(\mathbb{A}^n(\mathbb{F}_{q^d})).$$

Conclude that

$$A_k(\mathbb{A}^n(\mathbb{F}_q)) = 0, \quad k = 0, \dots, n-1.$$

Hint: Show there exists a point in $\mathbb{A}^n \setminus V$ over sufficiently large finite fields.

2)Let $f : X \to Y$ be a surjective morphism of smooth projective curves over $\mathbb{C}$. Give your own proof that

$$f_* \mathrm{div}(r) = \mathrm{div} N(r),$$

where $N : R(X) \to R(Y)$ is the norm.

3)Let $f : X \to Y$ be a flat morphism of irreducible varieties over K. Show that f is dominant.

4)Let $f : X \to Y$ be a flat morphism of irreducible varieties over K. Show that for each y in the image of f

$$\dim f^{-1}(y) = \dim X - \dim Y.$$

Hint: Prove this first for a generic point $y_0 \in Y$. For $y \in Y$ arbitrary, exhibit a smooth curve C and a morphism $g : C \to Y$ with image containing y and y_0. Use the fact that

$$X \times_Y C \to C$$

is flat.

5)Give an example of a *non-flat* morphism of varieties $f : X \to Y$ with

$$\dim f^{-1}(y) = \dim X - \dim Y$$

for each y in the image of f.