January 11, 2008

1)Let K be an infinite field. Give a detailed proof that affine space $\mathbb{A}^n(K)$ has vanishing Chow groups

$$A_k(\mathbb{A}^n(K)) = 0, \quad k = 0, \ldots, n-1.$$

2)Let K be an infinite field. Let $H \subset \mathbb{P}^n(K)$ be a hyperplane section of projective space. Show that each k-cycle of $\mathbb{P}^n(K)$ for $k < n$ is rationally equivalent to a cycle supported in H.

3)Show that any two lines $L_1, L_2 \subset \mathbb{P}^n(K)$ are rationally equivalent.

4)Let $f : X \hookrightarrow Y$ be a closed imbedding of algebraic varieties over K, i.e., X is a closed subvariety of Y. Show there is a well-defined push-forward homomorphism

$$f_* : A_k(X) \to A_k(Y).$$

Give a careful proof that f_* respects rational equivalence.

5)Consider the complex plane curve

$$C = \{[x, y, z] : x^3 + y^3 = z^3\} \subset \mathbb{P}^2(\mathbb{C})$$

and the points

$$p_1 = [0, 1, 1], \; p_2 = [1, 0, 1], \; p_3 = [1, -1, 0].$$

Show that $3[p_1] = 3[p_2] = 3[p_3]$ in $A_0(C)$. Can you find another point $q \in C$ with $3q = 3p_1$?