Math 465 Assignment 11: Due Friday, April 23

Throughout, we work over an algebraically closed field.
1)Let $Z \subset \mathbb{P}^n, n \geq 2$, be a finite nonempty subset, $\mathcal{I}_Z$ the ideal sheaf of Z, and $d > 0$ an integer.
a)Show that $H^1(\mathbb{P}^n, \mathcal{I}_Z(d)) \neq 0$ if and only if Z imposes fewer than $|Z|$ independent conditions on homogeneous forms of degree d.
b)What happens in the case $n = 1$? Is the assertion of part a) still true?
c)How do you interpret $\dim H^1(\mathbb{P}^n, \mathcal{I}_Z(d))$?

2) Let $\mathcal{F}$ be a coherent $\mathcal{O}_{\mathbb{P}^n}$-module. Show carefully that

$$H^q(\mathbb{P}^n, \mathcal{F}) = 0, q > n.$$

3)Let X be a affine variety with an infinite number of points. Show that $\Gamma(X, \mathcal{O}_X)$ is infinite dimensional.

4)Let $Z = \{[1,0,0],[1,0,1],[1,1,0],[1,1,1]\} \subset \mathbb{P}^2$ and let $\mathcal{I}_Z$ denote its ideal sheaf.
a)Let $J \subset k[x_0, x_1, x_2]$ denote the ideal of polynomials vanishing on Z. Show that

$$J = \langle x_1(x_1 - x_0), x_2(x_2 - x_0) \rangle .$$

b)Show there is an exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^2}(-4) \stackrel{\psi_2}{\to} \mathcal{O}_{\mathbb{P}^2}(-2)^2 \stackrel{\psi_1}{\to} \mathcal{I}_Z \to 0$$

where

$$\psi_1 = \begin{pmatrix} x_1(x_1 - x_0) & x_2(x_2 - x_0) \end{pmatrix} \quad \psi_2 = \begin{pmatrix} x_2(x_2 - x_0) \\ -x_1(x_1 - x_0) \end{pmatrix} .$$

c)Compute the dimensions of all the cohomology groups $H^i(\mathbb{P}^2, \mathcal{I}_Z(d))$.
d)For which values of d is $\mathcal{I}_Z(d)$ globally generated?

5)Let $P \in k[x_0, x_1, x_2]$ be irreducible and homogeneous of degree d, and $C = X(\langle P \rangle) \subset \mathbb{P}^2$ the plane curve where P vanishes. Compute the cohomology groups $H^i(\mathcal{O}_C)$.

6)Let C be the locus in $\mathbb{P}^3$ defined by the ideal

$$\left\langle x_0x_2 - x_1^2, x_0x_3 - x_1x_2, x_1x_3 - x_2^2 \right\rangle.$$

Show that $\dim \Gamma(\mathcal{O}_C(d)) = 3d + 1$ for $d \geq 0$.