Math 465 Assignment 3: Due Monday, September 12

1)Consider the polynomial

$$f = x^4 + x^2y^2 + y^3 - x^3 \in \mathbb{C}[x, y]$$

and the ideal

$$I = \langle f, \partial f/\partial x, \partial f/\partial y \rangle .$$

Compute the dimension of

$$\mathbb{C}[x, y]/I$$

as a complex vector space. Determine whether $x^5 \equiv y^5 \pmod{I}$.

2)Using the Buchberger algorithm, compute Gröbner bases for

$$\left\langle x_3 - x_1^5, x_2 - x_1^3 \right\rangle$$

with respect to both lexicographic order and graded reverse lexicographic order (defined in the lecture notes). Include all the relevant S-polynomial calculations. Which computation takes more effort?

Compute the normal form of $x_1x_2x_3$ with respect to each Gröbner basis.

3)Fix a monomial order $<$ on $k[x_1, \ldots, x_n]$ and a nonzero ideal $I \subset k[x_1, \ldots, x_n]$. A *reduced Gröbner basis* for I is a Gröbner basis $\{f_1, \ldots, f_r\}$ with the following additional properties:

1. $\mathrm{LT}(f_j) = \mathrm{LM}(f_j)$ for each j, i.e., the leading coefficient of f_j equals one;

2. for each i and j with $i \neq j$, no term of f_i is divisible by $\mathrm{LM}(f_j)$.

Show that I admits a unique reduced Gröbner basis.