due April 14: Do problems 14, 15a, 19c, 29 from section 17.1 and the following:

1)Let k be a field and $R = k[x_1, x_2, x_3]$ and $M = R/\langle x_1, x_2, x_3\rangle$. Compute the groups $\mathrm{Ext}^i_R(k, R)$ for each i.

2)

a. Compute $\mathrm{Ext}^1_{\mathbb{Z}}(\mathbb{Z}/4\mathbb{Z}, \mathbb{Z}/4\mathbb{Z})$.

b. Consider the extension

$$0 \to \mathbb{Z}/4\mathbb{Z} \xrightarrow{\alpha} \mathbb{Z}/16\mathbb{Z} \xrightarrow{\beta} \mathbb{Z}/4\mathbb{Z} \to 0$$

where $\alpha(n \pmod 4) = 4n \pmod{16}$ and $\beta(m \pmod{16}) = m \pmod 4$. Let $\eta \in \mathrm{Ext}^1_{\mathbb{Z}}(\mathbb{Z}/4\mathbb{Z}, \mathbb{Z}/4\mathbb{Z})$ be the corresponding class. Write down extensions corresponding to integer multiples of η.

3)Let L and N be R-modules. Show that the split extension

$$0 \to L \to L \oplus N \to N \to 0$$

gives rise to the class $0 \in \mathrm{Ext}^1_R(N, L)$.

4)Let R be a commutative ring with R-modules L, M, and N.

a. Show that composition gives a R-bilinear map

$$\begin{array}{rcl} \mathrm{Hom}_R(N, M) \times \mathrm{Hom}_R(M, L) & \to & \mathrm{Hom}_R(N, L) \\ (\phi, \psi) & \mapsto & \psi \circ \phi. \end{array}$$

b. Show there is a well-defined map

$$\mathrm{Ext}^1_R(N, M) \times \mathrm{Hom}_R(M, L) \to \mathrm{Ext}^1_R(N, L)$$

coming from the following recipe: Given an extension

$$0 \xrightarrow{\alpha} M \to W \xrightarrow{\beta} N \to 0$$

and a homomorphism

$$\psi : M \to L$$

consider the induced extension

$$0 \to M \oplus L \to W \oplus L \to N \to 0$$

where the first map corresponds to

$$\begin{pmatrix} \alpha & 0 \\ \psi & \mathrm{Id}_L \end{pmatrix}$$

and the second map is $\beta \oplus 0$. Regard M as a submodule of $M \oplus L$ and verify that the induced homomorphism $M \to W \oplus L$ is injective. Then we obtain

$$0 \to (M \oplus L)/M \to (W \oplus L)/M \to N \to 0$$

which yields an extension of N by L

$$0 \to L \to (W \oplus L)/M \to N \to 0.$$

c. **Optional Challenge:** Show that the pairing in (b) is R-bilinear.

d. Emulating the recipe of (b), exhibit a map

$$\mathrm{Hom}_R(N, M) \times \mathrm{Ext}^1_R(M, L) \to \mathrm{Ext}^1_R(N, L).$$