Here we elaborate the discussion on page 63 of Miranda:

Let $L \subset \mathbb{C}$ be a lattice and $\gamma$ a complex number such that $\gamma L = L$. Then one of the following holds:

1. $\gamma = \pm 1$ and $L$ is arbitrary;
2. $\gamma = \pm i$ and $L \sim \mathbb{Z} + \mathbb{Z}i$;
3. $\gamma = \exp(\pi i k/3), k = 1, 2, 4, 5$ and $L \sim \mathbb{Z} + \mathbb{Z}\exp(\pi i/3) = \mathbb{Z} + \mathbb{Z}\frac{1+\sqrt{-3}}{2}$.

Here the equivalence $L \sim L'$ means there exists an $\alpha \in \mathbb{C}$ so that

$$L' = \alpha L = \{\alpha \ell : \ell \in L\}.$$

Observe that $\gamma L = L$ if and only if $\gamma L' = L'$.
*proof:* Replacing $L$ by an equivalent lattice, we may assume

$$L = \mathbb{Z} + \mathbb{Z}\omega$$

where $\omega$ is a (non-real) complex number. Since $\gamma L \subset L$ we can write

$$\gamma = \gamma 1 = a_{11} + a_{21}\omega, \quad \gamma\omega = a_{12} + a_{22}\omega$$

where the $a_{ij}$ are integers. Note that

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

is the matrix for the $\mathbb{Z}$-linear transformation

$$\begin{array}{rcl} \mu_\gamma : L & \to & L \\ \ell & \mapsto & \gamma\ell. \end{array}$$

Since this is invertible, we have

$$\det(A) = a_{11}a_{22} - a_{12}a_{21} = \pm 1.$$

We can also interpret $\mu_\gamma$ as an $\mathbb{R}$-linear transformation

$$\begin{array}{rcl} \mathbb{R}^2 & & \mathbb{R}^2 \\ \| & & \| \\ \mu_\gamma : \mathbb{C} & \to & \mathbb{C} \\ z & \mapsto & \gamma z. \end{array}$$

Using the basis $\{1, i\}$, we get a matrix for $\mu_\gamma$

$$\begin{pmatrix} \mathrm{Re}(\gamma) & -\mathrm{Im}(\gamma) \\ \mathrm{Im}(\gamma) & \mathrm{Re}(\gamma) \end{pmatrix}$$

so that

$$\det(\mu_\gamma) = \mathrm{Re}(\gamma)^2 + \mathrm{Im}(\gamma)^2 = |\gamma|^2.$$

The determinant of a linear transformation does not depend on the choice of basis, so $\det(A) = \det(\mu_\gamma)$ and thus $|\gamma| = 1$.

Consider the characteristic polynomial of $A$

$$p_A(t) = \det(tI - A) = t^2 - \mathrm{tr}(A)t + \det(A) = t^2 - \mathrm{tr}(A)t + 1.$$

The Cayley Hamilton Theorem implies that $p_A(A) = 0$, so $p_A(\gamma)$ acts on $L$ by multiplication by zero and

$$\gamma^2 - \mathrm{tr}(A)\gamma + 1 = 0.$$

Taking complex conjugates

$$\bar{\gamma}^2 - \mathrm{tr}(A)\bar{\gamma} + 1 = 0.$$

Since $\gamma$ is imaginary, $\gamma$ and $\bar{\gamma}$ are distinct roots of $t^2 - \mathrm{tr}(A)t + 1$ and

$$\mathrm{tr}(A) = \gamma + \bar{\gamma} = 2\mathrm{Re}(\gamma).$$

Since $\gamma$ is on the unit circle, the only possibilities are

$$\mathrm{Re}(\gamma) = 0, \pm 1/2, \pm 1$$

whence

$$\gamma = \pm i, \frac{\pm 1 \pm \sqrt{-3}}{2}, \pm 1.$$

In the first instance we have

$$L = \mu_\gamma \mathbb{Z} = \mathbb{Z} + \mathbb{Z}i.$$

In the second we have

$$L = \mu_\gamma \mathbb{Z} = \mathbb{Z} + \mathbb{Z}\gamma = \mathbb{Z} + \mathbb{Z}\frac{1 + \sqrt{-3}}{2}.$$

Finally, $\gamma = \pm 1$ may act on any lattice $L$.