Problem set 12: due April 16

1)2)3) Exercises P-5, P-15, P-20 from Chapter 7 of the text.

Consider the Poincaré model of the hyperbolic plane, bounded by a Euclidean circle γ:

4)Let ℓ and m be P-lines containing an ideal point $\Omega \in \gamma$.
a)Show that ℓ is parallel to m. *Hint:* If ℓ and m are arcs of the Euclidean circles α and β, show that α and β intersect only at Ω.
b)Show that there is no P-line k perpendicular to both ℓ and m.
c)Conclude that ℓ and m are asymptotically parallel.

5)Let ℓ and m be parallel P-lines not containing a common ideal point. Show there is a P-line k perpendicular to both ℓ and m.

6)Let ℓ and m be distinct P-lines. Show there exists a P-line k so that Poincaré reflection across k takes ℓ to m.

7)Prove directly that the Poincaré model satisfies the first part of Betweenness Axiom B-4: Given a P-line ℓ and three points A, B, C not on ℓ, if AB and BC do not intersect ℓ then AC does not intersect ℓ. *Hint:* Without loss of generality, you may assume ℓ is a diameter of γ. Show that the P-segment AB meets ℓ iff A and B are on opposite sides of the Euclidean line containing ℓ.