Handout: Math 366

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Let $\mathcal{A}$ be an affine incidence plane and consider

$$\mathcal{A}^* = \mathcal{A} \cup \{\text{equivalence classes of parallel lines in } \mathcal{A}\}.$$

Precisely, we define

$$\begin{aligned} \text{points}(\mathcal{A}^*) &= \text{points}(\mathcal{A}) \cup \{ \text{ equivalence classes } [\ell] \text{ for } \ell \in \text{lines}(\mathcal{A})\} \\ \text{lines}(\mathcal{A}^*) &= \text{lines}(\mathcal{A}) \cup \{\ell_\infty\} \qquad \text{"the line at infinity"} \end{aligned}$$

with the following incidence relation between points($\mathcal{A}^*$) and lines($\mathcal{A}^*$):

1. For $p \in$ points($\mathcal{A}$): p is incident to $m \in$ lines($\mathcal{A}$) in $\mathcal{A}^*$ if and only if p is incident to m in $\mathcal{A}$; p is not incident to the line at infinity ℓ_∞.

2. For $[\ell]$ an equivalence class of parallel lines: $[\ell]$ is incident to $m \in$ lines($\mathcal{A}$) if and only if $m \in [\ell]$; $[\ell]$ is incident to the line at infinity ℓ_∞.

Theorem 0.1 *$\mathcal{A}^*$ is a projective plane and is called the* projective completion *of the affine plane $\mathcal{A}$. In particular, $\mathcal{A}^*$ satisfies the axioms of incidence geometry and the defining properties of projective planes.*

The proof of this is in the text, pages 59-61.