On sets of large exponential sums *

Shkredov I.D.

Let A be a set of residue classes modulo a positive integer N, and let $\alpha \in (0,1)$. What is the structure of the set $\mathcal{R}_\alpha(A)$ of those elements $u \in \mathbf{Z}_N$ such that the Fourier transform $\widehat{A}(u)$ of the set A at u satisfies $|\widehat{A}(u)| \geq \alpha |A|$? The set $\mathcal{R}_\alpha(A)$ is an analog of point spectrum of a dynamical system. Results in this direction were obtained by Freiman and Yudin in the 70th, Besser in 1999, Lev in 2002 and Green in 2004. Another important result was obtained by Chang who has applied it to sharpen the well–known theorem of Freiman on the structure of sets with small sumset. Recently, the author proved, in particular, that the equation $r_1 + r_2 = r_3 + r_4$ has at least $|\mathcal{R}_\alpha(A)|^{2+\varepsilon}$ solutions in $r_1, r_2, r_3, r_4 \in \mathcal{R}_\alpha(A)$ for a fixed absolute constant $\varepsilon > 0$. We are going to discuss some of the generalizations and applications of our results.

*This work was supported by RFFI grant no. 06-01-00383, President's of Russian Federation grant N 1726.2006.1 and INTAS (grant no. 03–51–5-70).