1. The concentration of a toxic chemical in the xy-plane at position (x, y) is given by $c(x, y) = e^{-x}(xy + 3 - \sin(x - y))$.

    (a) If an environmental worker is at $(x, y) = (1, 1)$ in what direction must they go to decrease the concentration as fast as possible?

    (b) In what directions could she move that would not change the concentration of the chemical?

2. Let $f(x, y, z) = (e^{-2xy}, x^2 - z^2 - 4x + \sin(x + y + z))$ and let $g : \mathbb{R}^2 \to \mathbb{R}$ such that $g(1, 0) = -1$ and $\nabla g(1, 0) = (1, -3)$.

    (a) Calculate the gradient of $g(f(x, y, z))$ at the point (0,0,0).

    (b) Find equation of tangent plane to the level set $g(f(x, y, z)) = -1$ at the point (0,0,0).

3. (a) Find extreme points of $f(x, y) = x^2 - 2x + y^2 - 1$ and classify them.

    (b) For same f, find the absolute max and min of f restriced to the curve $x^2 + y^2 = 1$.

4. Let $\mathbf{F}(x, y) = (2xy, x^2)$. Show the integral of $\mathbf{F}$ around the circumference of the square $[0, 1] \times [0, 1]$ in the xy-plane is zero by:

    (a) direct evaluation

    (b) showing $\mathbf{F}$ is a gradient vector field

    (c) Green's Theorem

5. Let W be the region in space under the graph of $f(x, y) = \cos y e^{1-\cos(2x)} + xy$ over the region in the xy plane bounded by the line $y = 2x$ the x axis, and the line $x = \pi/4$.

    (a) Find the volume of W.

    (b) Let $\mathbf{F}(x, y, z) = (5x, 5y, 5z)$ be the velocity field of a fluid in space. Calculate the rate at which fluid is leaving the region W (think someone's theorem...)

6. Compute the flux of the vector field $G(x, y, z) = (xy^2, yz^2 + y, zx^2 + 1)$ through the unit sphere oriented outward.

7. Let $c(t) = (1, -t^2, \cos t), t \in [0, \pi]$. Evaluate

$$\int_c \sin z dx - y^2 dy + 3xz dz.$$

8. Use Stokes' theorem to calculate $\int_C \mathbf{F} \cdot dS$ where $\mathbf{F}(x, y, z) = (x^2y^3 + y - \cos(y^3), x^3y^2 + sin(y^3) + x, z)$, and C is the circle $x^2 + y^2 = 1, z = 0$.