Math 212 - Practice Exam 2

May 25th, 2006

1. Let $f(x,y) = \frac{1}{2}x^2 - xy + \frac{1}{3}y^3$.
   (a) Find all critical points of $f(x,y)$.
   (b) Classify the critical points using the 2nd derivative test.
2. Let $g(x,y) = e^x \cos y$.
   (a) Approximate g using the second order Taylor polynomial around the point (0,0).
   (b) Use above to estimate $e^{-0.2}\cos 0.1$.
3. A tank is in the shape of a half-cylinder of radius 2 and height 3. It is situated in $\mathbb{R}^3$ by the following inequalities: $\sqrt{x^2+y^2} \leq 2, y \geq 0, 0 \leq z \leq 3$. The temperature at a point inside the tank is given by $T(x,y,z) = 2yz^2\sqrt{x^2+y^2}$.
   (a) What is volume of the tank? You can do this in multiple ways.
   (b) What is $\int\int\int_S T(x,y,z)dxdydz$ where S is the tank? (Hint: change to cylindrical coordinates).
   (c) Calculate the average temperature inside the tank, i.e. $\frac{answer(b)}{answer(a)}$.
4. Find the absolute maximum and minimum of $f(x,y) = x + y^2$ on the elliptic disk $x^2 + 3y^2 \leq 9$.
5. Set up the integral to calculate the volume enclosed by the surfaces $x^2 + y^2 + z^2 = 2$ and $x^2 + y^2 = z$. Evaluate for extra credit.
6. Let W be the region in space under the graph of $f(x,y) = \cos y e^{1-\cos 2x} + xy$, over the xy plane bounded by the line $y = 2x, y = 0, x = 0, x = \pi/4$. Calculate the volume of W.
7. Evaluate
$$\int_0^{\sqrt[6]{\pi/2}} \int_{x^2}^{\sqrt[3]{\pi/2}} x^3 \sin(y^3) dydx$$
. Hint: Change order of integration.