Math 212 - Practice Exam 1

May 19th, 2006

1. Let \( P = (1, 1, 1), Q = (-1, 0, 5), R = (2, 3, -2). \)
   (a) Find the equation of the plane through the three points \( P, Q \) and \( R. \)

2. Let \( f(x, y) = y - x^2. \)
   (a) Sketch and describe the level curves of \( f(x, y) = c, \) for \( c = 0, 1, 4. \)
   (b) Sketch and describe the \( x = 0 \) and \( y = 0 \) sections of the graph of \( f(x, y). \)
   (c) Sketch and describe the graph of \( f. \)

3. Let \( c(t) = (t, t^2, 1) \) be the equation for the position of a particle traveling through space.
   (a) Find the velocity of the particle at time \( t. \)
   (b) Find the equation for the tangent line to \( c(t) \) at time \( t = 1. \)

4. (a) Find the equation for the tangent plane of the graph of \( f \) at the point \( (x_0, y_0, f(x_0, y_0)) \) for \( f(x, y) = (xe^y)^8 \) and \( (x_0, y_0) = (1, 0). \)
   (b) Use the tangent plane found above to approximate \((0.99e^{0.02})^8. \)

5. A bug finds itself in a toxic environment. The toxicity level is given by \( T(x, y) = 2x^2 - 4y^2. \) The bug is at \((-1, 2). \) In what direction should it move to lower the toxicity the fastest.

6. Give a careful statement of the general form of the chain rule. Let \( f(x, y) = x^2 + y \) and \( h(u) = (\sin 3u, \cos 8u). \) Let \( g(u) = f(h(u)). \) Compute \( \frac{dg}{du} \) at \( u = 0 \) directly and by using the chain rule.

7. In what direction is the directional derivative of \( f(x, y) = (x^2 - y^2)/(x^2 + y^2) \) at \((1, 1) \) equal to zero?