Math 212-4 - Practice/Review Questions for Exam 2

March 6, 2007

1. Make indicated change of variables, but do not evaluate.

   (a) $\int_0^1 \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \sqrt{x^2+y^2}dxdydz$, cylindrical coordinates

   (b) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} xyzdzdxdy$, cylindrical coordinates

   (c) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2dzdxdy$, spherical coordinates

   (d) $\int_0^1 \int_0^{\pi/4} \int_0^{2\pi} \rho^3 \sin 2\phi d\theta d\phi d\rho$

2. Evaluate
$$\int_0^1 \int_{\arctan(y)}^{\pi/4} \sec^5(x)dxdy$$

3. Write the iterated integral
$$\int_0^1 \int_{1-x}^1 \int_x^1 f(x,y,z)dzdydx$$
as an equivalent integral written in two other possible orders of integration.

4. Find the volume of the solid between the paraboloid $z = 3x^2 + 3y^2$ and $z = 12 - 3x^2 - 3y^2$.

5. Find the volume of the solid region (ice cream cone) that lies inside the sphere $x^2 + y^2 + z^2 = z$ and above the cone $z^2 = x^2 + y^2, z \geq 0$.

6. Show that $\mathbf{c}(t) = (t^{-3}, e^t, t^{-1})$ is a flow line of the vector field $\mathbf{F}(x,y,z) = (-3z^4, y, -z^2)$.

7. Compute divergence and curl for the vector field $\mathbf{F}(x,y,z) = (y,z,x)$ at the point $(1,1,1)$.

8. Calculate the arc length of the curve defined by $x = y^3 = z^2 + 1$ from the $x = 0$ to $x = 1$.