Math 212-4 - Practice/Review Questions for Exam 2

March 6, 2007

1. Make indicated change of variables, but do not evaluate.
   (a) \[ \int_{0}^{1} \int_{-1}^{1} \int_{\sqrt{1-y^2}}^{\sqrt{1-y^2}} \sqrt{x^2+y^2} \, dx \, dz \, dy, \text{cylindrical coordinates} \]
   (b) \[ \int_{-1}^{1} \int_{\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} xy \, dz \, dx \, dy, \text{cylindrical coordinates} \]
   (c) \[ \int_{-\sqrt{2}}^{\sqrt{2}} \int_{\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} z^2 \, dz \, dx \, dy, \text{spherical coordinates} \]
   (d) \[ \int_{0}^{\pi/4} \int_{0}^{2\pi} \int_{0}^{\rho} \rho^3 \sin 2\phi \, d\phi \, d\rho \, d\rho \]

2. Evaluate
   \[ \int_{0}^{1} \int_{\arctan(y)}^{\pi/4} \sec^5(x) \, dx \, dy \]

3. Write the iterated integral
   \[ \int_{0}^{1} \int_{1-x}^{1} \int_{x}^{1} f(x, y, z) \, dz \, dy \, dx \]
   as an equivalent integral written in two other possible orders of integration.

4. Find the volume of the solid between the paraboloid \( z = 3x^2 + 3y^2 \) and \( z = 12 - 3x^2 - 3y^2 \).

5. Find the volume of the solid region (ice cream cone) that lies inside the sphere \( x^2 + y^2 + z^2 = z \) and above the cone \( z^2 = x^2 + y^2 \), \( z \geq 0 \).

6. Show that \( c(t) = (t^{-3}, e^t, t^{-1}) \) is a flow line of the vector field \( \mathbf{F}(x, y, z) = (-3z^4, y, -z^2) \).

7. Compute divergence and curl for the vector field \( \mathbf{F}(x, y, z) = (y, z, x) \) at the point \( (1, 1, 1) \).

8. Calculate the arc length of the curve defined by \( x = y^3 = z^2 + 1 \) from the \( x = 0 \) to \( x = 1 \).