Math 212-4 - Practice/Review Questions for Exam 1

February 1, 2007

1. For vectors $\mathbf{a}, \mathbf{b}$, know definitions of $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$, $||\mathbf{a}||$, and geometric meaning.

2. Know equations for line, plane in space.

3. Sketch the level sets and sections for the function $f(x,y) = e^{-x^2-2y^2}$ and describe the graph of f.

4. Let $f(x,y) = x^2+y, \mathbf{h}(u) = (\sin 3u, \cos 8u)$. Let $g(u) = f(\mathbf{h}(u))$. Compute $\mathbf{D}g$ using the chain rule and by direct substitution. If you know $\mathbf{h}(0)$ and $\mathbf{h}'(0)$, can you find $\mathbf{D}(f(\mathbf{h}(0)))$?

5. Classify critical points of $f(x,y) = \frac{\sin(\pi x)}{1+y^2}$.

6. Find the extema of $f(x,y) = 3x + 2y$, subject to $2x^2 + 3y^2 = 3$.

7. Find the absolute max and min of $f(x,y) = x^2 + y^2 - x - y + 1$ on the unit disk, i.e. of $D = \{(x,y)|x^2 + y^2 \leq 1\}$.

8. Show the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8abc}{3\sqrt{3}}$. (Hint: Use Lagrange multipliers)

9. Find the plane tangent to the surface $z = x^2 + y^2$ at $(1,-2,5)$. Explain geometric significance, for this surface, of the gradient of $f(x,y) = x^2+y^2$.

10. A bug finds itself in a toxic environment. The toxicity level is given by $T(x,y) = 2x^2 - 4y^2$. The bug is at $(-1,2)$. In what direction should it move to lower the toxicity the fastest, and what is the rate.

11. In what direction is the directional derivative of $f(x,y) = (x^2-y^2)/(x^2+y^2)$ at $(1,1)$ equal to zero? What about an arbitrary point (x_0,y_0) in the first quadrant?