Math 212-4 - Exam 2

April 12, 2007

Instructions: You have **2 hours** to complete the exam. Write the time you start and end on the exam, along with the honor code. You may NOT use any books, notes, calculator, or other people. Do not discuss the exam with anyone except your instructor until all exams are turned in.

There are 7 questions, plus one bonus. Show all work to receive full credit. Partial credit will be given, so it is best to turn in all work. Indicate final answer clearly. The exam will be due **Thursday, March 22nd, 11 am**. No late exams will be accepted. Good luck!

Formulas you can use:

1. Cylindrical coordinates: $x = r\cos\theta, y = r\sin\theta, z = z$, $0 \le r, 0 \le \theta \le 2\pi$.
2. Spherical coordinates: $x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\phi$, $0 \le \rho, 0 \le \phi \le \pi, 0 \le \theta \le 2\pi$.

1. (15 points) Evaluate

$$\int_0^{27}\int_{y^{1/3}}^{3} x^{-4}y^2e^{x^6}\,dxdy$$

2. (15 points) Evaluate $\int\int_T x\sin(y^3)dA$ where T is the triangle with vertices $(0,0),(1,1),(0,1)$.

3. (15 points) Consider the region D bounded between the circle $x^2+y^2=1$ and $x^2+y^2=4$, $x\geq 0, y\geq 0$.

   (a) Write the double integral of a general function $f(x,y)$ over the region D as an iterated integral over x and y.

   (b) Write an equivalent integral after changing to polar coordinates

   (c) Calculate $\int\int_D \frac{\sin(\pi\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}dA$.

4. (15 points) Find the volume of the region enclosed by both the sphere $x^2+y^2+z^2=1$ and the cylinder $x^2+y^2=1/2$.

5. (15 points) Evaluate

$$\int\int\int_W \frac{z^2}{x^2+y^2+z^2}dxdydz$$

   where W is the solid "shell" bounded by the two spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$

6. (15 points) Let f be a real-valued, differentiable function of one variable. For each point (x,y,z), define $\mathbf{F}(x,y,z)=(-y*f(x^2+y^2), x*f(x^2+y^2), 0)$.

   (a) Calculate divergence of $\mathbf{F}$

   (b) Calculate curl of $\mathbf{F}$

   (c) What equation must $g(t)$ satisfy for $\mathbf{c}(t)=(\cos(g(t)),\sin(g(t)))$ to be a flow line for $\mathbf{F}$.

7. (10 points) Consider the curve defined by $z^6=x^3=(3/4y)^4$, between $x=0$ and $x=1$. Find the arc length by parameterizing the curve and evaluating an appropriate integral.

8. (Bonus) Find the volume of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\leq 1.$$

   (Hint: Change coordinates twice, first by a suitable scaling of each variable)