Math 212-4 - Exam 1

February 8, 2007

Instructions: You have **2 hours** to complete the exam. Write the time you start and end on the exam, along with the honor code. You may NOT use any books, notes, calculator, or other people. Do not discuss the exam with anyone except your instructor until all exams are turned in.

There are 6 questions. Show all work to receive full credit. Partial credit will be given, so it is best to turn in all work. Indicate final answer clearly. The exam will be due **Tuesday, February 13th, 11 am**. Good luck!

1. (1 pt each) Answer T (true), F (false), or circle correct answer; $\mathbf{a}, \mathbf{b}$ are vectors and $\mathbf{a}, \mathbf{b} \neq \mathbf{0}$.

   (a) $\mathbf{a} \cdot \mathbf{b}$ is a vector
   (b) $\mathbf{a} \times \mathbf{b}$ is a vector
   (c) $\mathbf{a}, \mathbf{b}$ are both perpendicular to $\mathbf{a} \times \mathbf{b}$
   (d) $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$.
   (e) If $\mathbf{a} \cdot \mathbf{b} = 0$, then $\mathbf{a}, \mathbf{b}$ are (parallel, perpendicular, neither)
   (f) if $\mathbf{a} \times \mathbf{b} = 0$, then $\mathbf{a}, \mathbf{b}$ are (parallel, perpendicular, neither)
   (g) Equations $y = -2x + 3$ and $\mathbf{l}(x) = (t - 1, -2t + 4), t \in \mathbb{R}$ represent the same line.
   (h) A line is given by $\mathbf{l}(t) = (1 + t, 3 - 2t, 8t)$, and a plane $\mathbf{P}$ by $z = -2x - 5y + 17$; does $\mathbf{l}(t)$ (lie in $\mathbf{P}$, intersect $\mathbf{P}$ in a point, neither)?
   (i) If $\mathbf{a}, \mathbf{b}$ are parallel vectors, then $|\mathbf{a} \cdot \mathbf{b}| = ||\mathbf{a}||||\mathbf{b}||$
   (j) $||\mathbf{a}|| = \mathbf{a} \cdot \mathbf{a}$

2. (4 pt each)

   (a) Sketch the level sets of the function $f(x, y) = 9 - x^2 - 3y^2$.
   (b) Sketch the $x = 0, y = 0$ sections of the graph of f.
   (c) Sketch/describe the graph of f.
   (d) Compute ∇f, and explain the geometric significance in relation to the first part of the problem.
   (e) Find the equation for the tangent plane of f at the point $(1, 1, 5)$.

3. (5 pt each) $f(x, y, z) = xy + \cos(x^2 + z^2), \mathbf{c}(t) = (t \sin t, t, t \cos t)$.

   (a) Find $\mathbf{D}(f(\mathbf{c}(t))) = \frac{d}{dt}(f \circ \mathbf{c}(t))$.
   (b) If $\mathbf{g} : \mathbb{R} \to \mathbb{R}^3$, $\mathbf{g}(0) = (0, 1, 0), \mathbf{g}'(0) = (1, 1, 1)$. Find $\mathbf{D}(f(\mathbf{g}(0)))$.

4. (5 pt each) The pressure $P(x, y)$ at a point (x, y) on a metal membrane is given the function $P(x, y) = 100e^{-x^2 - 2y^2}$.

   (a) Find the rate of change (directional derivative) of the pressure at the point $p = (0, 1)$ in the direction $v = (1, 1)$.
   (b) In what direction away from the point p does the pressure increase most rapidly? Decrease most rapidly?
   (c) What is the maximum rate of increase of pressure at p
   (d) Locate the direction(s) at p in which the rate of change of pressure is zero.

5. (20 pt) Find the absolute max and min of the function $f(x, y) = 2x^2 + 2y^2 - x + y$ on the unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$.

6. (20 pt) Find the volume and dimensions of the largest rectangular parallelepiped (prism) that can be inscribed in the sphere $x^2 + y^2 + z^2 = 1$