

Midterm, due 3/10 in class

You have 2 hours for this exam. You are not allowed to use your book, your notes, your calculator, etc.

1. Assume we have a commutative diagram of abelian groups

$$\begin{array}{ccccccccc} 0 & \rightarrow & A & \rightarrow & G & \rightarrow & B & \rightarrow & 0 \\ & & \downarrow f & & \parallel \text{id} & & \downarrow g & & \\ 0 & \rightarrow & A' & \rightarrow & G & \rightarrow & B' & \rightarrow & 0, \end{array}$$

where the vertical map in the middle is the identity and where g is an injection. Show that f is surjective.

2. Let X be the result of identifying the north pole and the south pole of S^2 . Compute the homology of X .
3. Let X be a CW-complex and $P : Y \rightarrow X$ a two-fold covering. We know from covering theory that $p_* : \pi_1(Y) \rightarrow \pi_1(X)$ is injective. Is it true that $p_* : H_n(Y) \rightarrow H_n(X)$ is injective for all n ? Justify your answer.

4. Assume we know that

$$\begin{aligned} H_i(X; \mathbb{Z}) &= 0, & \text{for } i > 3 \\ H_3(X; \mathbb{Z}) &= \mathbb{Z}/2 \oplus \mathbb{Z} \\ H_2(X; \mathbb{Z}) &= \mathbb{Z}/6 \oplus \mathbb{Z}/7 \\ H_1(X; \mathbb{Z}) &= \mathbb{Z}/4 \oplus \mathbb{Z}^2 \\ H_0(X; \mathbb{Z}) &= \mathbb{Z}. \end{aligned}$$

Compute $H_*(X; \mathbb{Z}/2)$.

5. Let $0 \rightarrow A \xrightarrow{i} B \xrightarrow{p} C \rightarrow 0$ be an exact sequence of abelian groups and G another abelian group. Show that there exists a long exact sequence

$$\begin{array}{ccccccccc} 0 & \rightarrow & \text{Tor}(A, G) & \xrightarrow{i_*} & \text{Tor}(B, G) & \xrightarrow{p_*} & \text{Tor}(C, G) & \rightarrow & \\ & & \rightarrow & A \otimes G & \xrightarrow{i_*} & B \otimes G & \xrightarrow{p_*} & C \otimes G & \rightarrow 0. \end{array}$$