Homework 9, due Friday 3/24

1. (8 points) Let $R = \mathbb{Q}[t^{\pm 1}]$. Let A be a finitely generated R–module.
   (a) Compute $\mathrm{Tor}^i_R(A, R/(t^2 - 1))$ for all $i = 0, 1, 2, \ldots$.
   (b) Compute $\mathrm{Ext}^i_R(A, R)$ for all $i = 0, 1, 2, \ldots$.

   You can use the fact that R is a PID.

2. (12 points) Let X be the Klein bottle. Give X a CW–complex structure with one 0-cell, two 1–cells and one 2–cell.
   (a) Compute $H_*(X; \mathbb{Z}/n)$ for any $n \in 0, 1, 2, \ldots$.
   (b) Compute $H^*(X; \mathbb{Z}/n)$ for any $n \in 0, 1, 2, \ldots$ using the CW–complex structure.
   (c) Compute $H^*(X; \mathbb{Z}/n)$ for any $n \in 0, 1, 2, \ldots$ using the computation of $H_*(X; \mathbb{Z}/n)$ and the Universal Coefficient Theorem.

3. (10 points) Let X be a topological space. Denote the suspension of X by ΣX. Recall that

$$\Sigma X = X \times [-1, 1]/X \times 1, X \times -1.$$

   (a) Use Meyer–Vietoris sequences to compute $H_*(\Sigma X)$ and $H^*(\Sigma X)$ in terms of the homology and cohomology groups of X.
   (b) Let $X = S^n$ and $f : S^n \to S^n$ a map of degree d. Note that $\Sigma X \cong S^{n+1}$. Show that there exists an induced map $\Sigma f : \Sigma S^n \to \Sigma S^n$ such that $\deg(\Sigma f) = d$.