Homework 4, due Friday 2/10

1. Assume we have a commutative diagram

$$\begin{array}{ccccccccc} A & \xrightarrow{i} & B & \xrightarrow{j} & C & \xrightarrow{k} & D & \xrightarrow{l} & E \\ \downarrow a & & \downarrow b & & \downarrow c & & \downarrow d & & \downarrow e \\ A' & \xrightarrow{i'} & B' & \xrightarrow{j'} & C' & \xrightarrow{k'} & D' & \xrightarrow{l'} & E', \end{array}$$

   furthermore assume that the horizontal maps form an exact sequence and assume that the vertical maps a, b, d, e are isomorphisms. Show that c is an isomorphism as well.
   This goes under the name "Five–Lemma" and can be found in pretty much any book in this field, but do it yourself. Happy diagram chasing.

2. At a recent conference scientists tried to introduce a new scientific world time, i.e. to each point at a specific moment in time an hour $h(p) \in [0, 24]$ should be associated. The required specifications were:

   (a) At the equator, at a point with longitude $l \in [-180, 180]$ the time is $h = \frac{l+180}{15}$.
   (b) The time should be continuous (using the identification $0 = 24$).

   Did the scientists succeed?

3. The singular chain

$$\begin{array}{rcl} \sigma : \Delta^0 & \to & S^1 \\ (t_0, t_1) & \mapsto & e^{2\pi i t_0} \end{array}$$

   gives a generator g of $H_1(S^1)$ (you don't have to show this). Show that the singular chain

$$\begin{array}{rcl} \sigma' : \Delta^0 & \to & S^1 \\ (t_0, t_1) & \mapsto & e^{-2\pi i t_0} \end{array}$$

   represents $-g$. Do this by explicitly finding an element of $C_2(S^1)$ whose boundary equals $\sigma + \sigma'$.

4. Show that $H_n(S^n)$ for n even can never be generated by a singular chain (i.e. a generator is always a linear combination).

5. Does there exist a map $S^n \to S^n$ with no fixed points which is not surjective?

6. An extension of the excision theorem: Assume we have $A \subset Y \subset X$, and assume that there exists an open neighborhood R of Y in X such that $R \setminus A$ deformation retracts to $Y \setminus A$. Then the inclusion $(X \setminus A, Y \setminus A) \to (X, Y)$ induces an isomorphism

$$H_n(X \setminus A, Y \setminus A) \to H_n(X, Y).$$