

## Homework 3, due Friday 2/3

1. Assume  $A$  is an abelian group which fits into an exact sequence

$$0 \rightarrow \mathbb{Z}/p \rightarrow A \rightarrow \mathbb{Z}/q \rightarrow 0,$$

where  $p, q$  are prime numbers.

- (a) For which  $p, q$  is  $A$  uniquely determined?  
 (b) Determine the possible answers for  $A$ .
2. Let  $\mathcal{F}$  be a field. Assume we have an exact sequence

$$0 \rightarrow \mathcal{F}^{n_k} \rightarrow \mathcal{F}^{n_{k-1}} \rightarrow \dots \rightarrow \mathcal{F}^{n_1} \rightarrow \mathcal{F}^{n_0} \rightarrow 0$$

of homomorphisms.

- (a) Show that
- $$0 \rightarrow \mathcal{F}^{n_k} \rightarrow \mathcal{F}^{n_{k-1}} \rightarrow \mathcal{F}^{n_2} \dots \rightarrow \text{Im}(\mathcal{F}^{n_2} \rightarrow \mathcal{F}^{n_1}) \rightarrow 0$$
- is exact.
- (b) Show that  $\sum_{i=0}^k (-1)^i n_i = 0$ .
3. Let  $0 \rightarrow A_* \rightarrow B_* \rightarrow C_* \rightarrow 0$  be a short exact sequence of complexes. Show that if  $A_*$  and  $B_*$  are exact, then  $C_*$  is exact as well.
4. Read: [http://en.wikipedia.org/wiki/Snake\\_lemma](http://en.wikipedia.org/wiki/Snake_lemma) and give a proof of the Snake lemma (for abelian groups).
5. Let  $0 \rightarrow A_* \xrightarrow{i} B_* \xrightarrow{\pi} C_* \rightarrow 0$  be a short exact sequence of complexes. Proof the following inclusions without looking at the book (except for reading the definition of the connecting homomorphism):
- (a)  $\text{Ker}\{\partial : C_n \rightarrow C_{n-1}\} \subset \text{Im}\{\pi : B_n \rightarrow C_n\}$ .  
 (b)  $\text{Ker}\{i : A_n \rightarrow B_n\} \subset \text{Im}\{\partial : C_{n+1} \rightarrow A_n\}$ .
6. Let  $I = [0, 2]$ . Show that  $H_n(I, \{0\} \cup \{1\} \cup \{2\}) = 0$  for  $n > 1$  and  $n = 0$  and that  $H_n(I, \{0\} \cup \{1\} \cup \{2\}) \cong \mathbb{Z}^2$ .