

Homework 2, due Friday 1/27

1. Let X be any set with the topology that only the empty set and X are open sets. Show that with this topology $H_0(X) \cong \mathbb{Z}$.
2. We say that two complexes (D_*, ∂) and (C_*, ∂) are homotopy equivalent if there exist chain maps $f_* : C_* \rightarrow D_*$ and $g_* : D_* \rightarrow C_*$ such that $f_n \circ g_n$ and $g_n \circ f_n$ are chain homotopic to the identity map for any n . Show that $H_n(C_*) \cong H_n(D_*)$.

3. Consider the complex

$$0 \rightarrow \mathbb{Z} \xrightarrow{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \mathbb{Z}^2 \xrightarrow{\begin{pmatrix} 1 & 1 \end{pmatrix}} \mathbb{Z} \rightarrow 0.$$

Show that this complex is homotopy equivalent to the zero complex.

4. p. 132, problem 11.
5. Let $X = S^1 \times S^1$ be the torus.
 - (a) We know that $H_1(X) = \mathbb{Z}^2$ because $\pi_1(X) = \mathbb{Z}^2$. What do you think are two generators for $H_1(X)$? (no justification required)
 - (b) Give an example of an element in $\text{Ker}\{\partial : C_2(X) \rightarrow C_1(X)\}$. (the answer doesn't have to be 100% precise)
6. Let $X \rightarrow Y$ be an injective (respectively surjective) map. Does it follow that $H_1(X) \rightarrow H_1(Y)$ is injective (respectively surjective). If yes, then prove it. If not, then give a counter example. Perhaps it helps to first think of π_1 .