
Homework 12, due Friday 4/24

1. Recall that the Euler characteristic of a finite dimensional CW-complex X is defined as

$$\chi(X) = \sum_i (-1)^i \dim(H_i(X; \mathbb{Q})).$$

Show that $\chi(S^1 \times X) = 0$ for any X .

2. Compute $H_*(X; \mathbb{Z})$ where $X = S^1 \times \cdots \times S^1$ is the product of k copies of S^1 .
3. Determine the cohomology ring $H^*(S^1; \mathbb{Z})$ (as you perhaps immediately noticed, there's not much to say). Now use this and the Künneth theorem to determine the cohomology ring of the torus (and then we finally have a formal argument for it).
4. Hatcher p. 229, problem 6.
5. Hatcher p. 229, problem 11.