
Homework 11, due Monday 4/10

1. Hatcher p. 228: Problem 1 (several ideas perhaps: (1) Look at the relative homology and cohomology on the left picture, how does it relate to the right hand picture? (2) Study the homology and cohomology of both spaces and the map between them. (3) Finally study the cup product).
2. Hatcher p. 228: Problem 2. (What is $H^i(X, A)$ where $i > 0$ and A is contractible?)
3. Use the previous problem to show that $\mathbb{R}P^n$ can not be covered by $n - 1$ contractible open sets.
4. Hatcher p. 229: Problem 3 (a)
5. Show that there exists a space X with $H_i(X; \mathbb{Z}/2) = \mathbb{Z}/2$ for $i = 0, 1, 2$ and $H_i(X; \mathbb{Z}/2) = 0$ for $i > 2$ which is not homeomorphic to $\mathbb{R}P^2$.
6. Hatcher p. 229: Problem 7 (what about the fundamental group of the two spaces?)