Homework 1, due Friday 1/20

1. Let f be a map between topological spaces X and Y.
   (a) Show that the map $C_n(X) \to C_n(Y)$ which was defined in class induces a map $f_* : H_n(X) \to H_n(Y)$.
   (b) Show that $(f \circ g)_* = f_* \circ g_*$ and $\mathrm{id}_* = \mathrm{id}$.
2. Let X be a topological space. Show that $H_*(X) \neq 0$ ONLY using the computation of the homology of a point and using the property that $(f \circ g)_* = f_* \circ g_*$ and $\mathrm{id}_* = \mathrm{id}$.
3. Let X be any set with the topology that only the empty set and X are open sets. Show that with this topology $H_0(X) \cong \mathbb{Z}$.
4. Let $C_2 = \mathbb{Z}, C_1 = \mathbb{Z}^3, C_0 = \mathbb{Z}, C_i = 0$ for $i \in \mathbb{Z} \setminus \{0, 1, 2\}$. Let $\partial_2 : C_2 \to C_1$ be the map given by the matrix $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ and let $\partial_1 : C_1 \to C_0$ be the map given by the matrix $\begin{pmatrix} 2 & 0 & 4 \end{pmatrix}$. Furthermore let $\partial_i : C_i \to C_{i-1}$ be the zero map for $i \neq 1, 2$.
   (a) Show that $\partial_i \circ \partial_{i+1} = 0$ for all i.
   (b) Determine the group $\mathrm{Ker}(\partial_i)/\,\mathrm{Im}(\partial_{i+1})$ for all i.