MATH 445: Algebraic Topology **Spring 2006**

Final exam, due 5/8

You have 2 hours for this exam. You are not allowed to use your book, your notes, etc. All manifolds are assumed to be connected and oriented. You can always use the results of the other problems.

1. Compute $\text{Ext}(\mathbb{Z}/n, \mathbb{Z})$.

2. Let M be a closed n–manifold. Show that $H_{n-1}(M;\mathbb{Z})$ is $\mathbb{Z}$–torsion free. (Hint: compute $H^n(M;\mathbb{Z})$ in two different ways).

3. Let N and M be closed n–manifolds. Assume that $f : N \to M$ is a degree one map, i.e. the induced map $f_* : H_n(N;\mathbb{Z}) \to H_n(M;\mathbb{Z})$ is an isomorphism. Give a careful argument to show that the induced map $f^* : H^n(M;\mathbb{Z}) \to H^n(N;\mathbb{Z})$ is an isomorphism as well.

4. Show that there exists no degree one map from S^4 to $\mathbb{C}P^2$.

5. Let M be a compact n–manifold with connected boundary ∂M. Show that the map $H_{n-1}(\partial M;\mathbb{Z}) \to H_{n-1}(M;\mathbb{Z})$ induced by the inclusion $\partial M \subset M$ is the zero map.

6. Let M_1 and M_2 be closed oriented n–manifolds. Let $B_i \subset M_i$ be closed balls. Then the connected sum of M_1 and M_2 is defined to be

$$M_1 \# M_2 = M_1 \setminus \text{int}(B_1) \cup_{\partial B_1 = \partial B_2} M_2 \setminus \text{int}(B_2).$$

   Here ∂B_1 and ∂B_2 get identified via an orientation reversing homoeomorphism. Show that $\tilde{H}_k(M_1 \# M_2;\mathbb{Z}) \cong \tilde{H}_k(M_1;\mathbb{Z}) \oplus \tilde{H}_k(M_2;\mathbb{Z})$.
   Note: The homeomorphism type of $M_1 \# M_2$ is independent of the choice of B_1 and B_2. This fact, and also the orientations are completely irrelevant for the solution of the problem.