

1. Find the arc length of  $\mathbf{c}(t) = \left(t + 1, \frac{2\sqrt{2}}{3}t^{3/2} + 7, \frac{1}{2}t^2\right)$  where  $1 \leq t \leq 2$ .
2. Evaluate
  - (a)  $\operatorname{div} \mathbf{G}(x, y)$  where  $\mathbf{G}(x, y) = (-x, -y)$
  - (b)  $\operatorname{curl} \mathbf{V}$  where  $\mathbf{V}(x, y, z) = (x + y, y - x, z^2 - 3x)$ .
3. Evaluate
  - (a)  $\int_{\mathbf{c}} f ds$  where  $f(x, y, z) = \frac{x(x+y)}{y+z}$  and  $\mathbf{c}(t) = (t, \frac{1}{2}t^2, t)$  for  $1 \leq t \leq 2$ .
  - (b)  $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$  where  $\mathbf{F} = (x, -y, z)$  and  $\mathbf{c}(t) = (\cos t, \sin t, 2t)$  for  $t \in [0, \pi]$ .
  - (c)  $\int_{\mathbf{c}} ye^z dx + xe^z dy + xye^z dz$  where  $\mathbf{c}(t) = (\sin(t^3 + t/5), e^{\cos(t)}, 27t/e^{2t})$ , for  $0 \leq t \leq 3$ .
4. (a) Let  $T(x, y, z) = 3x^2 + 3z^2$  be the temperature in  $\mathbb{R}^3$ . Compute the heat flux

$$\int_S -\nabla T \cdot d\mathbf{S}$$

across the surface  $S$  given by  $x^2 + z^2 = 2$  and  $0 \leq y \leq 2$ .

- (b) Find  $\int_S z^2 dS$  where  $S$  is the unit sphere.
5. Give a parametrization for the shell (outer surface) of the following grain silo: the ‘walls’ are an elliptical-based cylinder of height 5 and base  $x^2 + \frac{y^2}{4} = 1$ , the roof is half of an ellipsoid that fits perfectly on the top of the elliptical cylinder, and has height 3, and don’t forget to include the elliptical base.

## Solutions

1.  $5/2$
2. (a)  $-2$   
(b)  $(0, 3, -2)$
3. (a)  $\frac{1}{2}(6^{3/2} - 3^{3/2})$   
(b)  $2\pi^2$   
(c)  $\sin\left(\frac{138}{5}\right) e^{\left(\cos 3 + \frac{81}{e^6}\right)}$
4. (a)  $-48\pi$  (sign is important)  
(b)  $\frac{4\pi}{3}$
5. Base:  $x = r \cos \theta, y = 2r \sin \theta, z = 0$  for  $r \in [0, 1], \theta \in [0, 2\pi]$ .  
Walls:  $x = \cos \theta, y = 2 \sin \theta, z = u$  for  $\theta \in [0, 2\pi], u \in [0, 5]$ .  
Roof:  $x = \cos \theta \sin \phi, y = 2 \sin \theta \sin \phi, z = 3 \cos \phi + 5$  for  $\theta \in [0, 2\pi], \phi \in [0, \pi/2]$ .