Homework 5, due Friday 2/17

1. p. 191, problems 2, 8 (a), 9.
2. Let $f(x, y, z) = x + y^2 + z^3 + xy$.
   (a) Find the first order Taylor polynomial at the point $(0, 0, 0)$.
   (b) Find the second order Taylor polynomial at the point $(0, 0, 0)$.
   (c) *Guess* the second order Taylor polynomial of $f(x, y, z) = x^3 + 2y + z^3 + yz$ at the point $(0, 0, 0)$.
3. Let $f(x, y) = xe^{x+y-1}$.
   (a) Find the first order Taylor polynomial at the point $(1, 0)$. Use this to give an estimate for $f(1.1, 0.1)$.
   (b) Find the second order Taylor polynomial at the point $(1, 0)$. Use this to give an estimate for $f(1.1, 0.1)$.
   (c) Calculate $f(1.1, 0.1)$ with a calculator, compare with the previous estimates.
4. p. 222, problems 5, 7
5. Let $f(x, y) = \sin(x^2 + y^2)$.
   (a) Compute the second order Taylor polynomial T_2 at the point $(0, 0)$.
   (b) Show that $(0, 0)$ is a critical point for f and T_2.
   (c) Show that f and T_2 have the same discriminant at $(0, 0)$.
   (d) Show that f and T_2 a local minimum at $(0, 0)$.
6. p. 224, problems 33, 34. Do NOT use Lagrange multipliers, but use the method of Section 3.3 instead, to find the maximum and minimum on the boundary.
7. Why does $f(x, y) = x^4 + y^4$ have a local minimum at $(0, 0)$?