Homework 4, due Friday 2/10

1. p. 149, problems (1), (3), (5), (14)
2. p. 150, problems (15), (17)
3. p. 159, problem (5), (a)–(c). First compute $\frac{d}{dt}f(\mathbf{c}(t))$ using the chain rule. Then write $f(x(t), y(t))$ as a t–function by plugging in and then compute the derivative as an ordinary one variable function. Compare the results.
4. p. 160, problem 12 (for 12 (b) use the tangent line)
5. p. 171, problems (1), (2) (a), (b)
6. p. 171, problem (6) (a), (b) and determine the direction of fastest increase at the point $(1, 1, 1)$.
7. Let $f(x, y) = x^2 + y^2$.
   (a) Sketch the level surfaces for $c = 0, 1, 4, 9$.
   (b) Sketch the gradient for the points $(0, 0), (0, 1), (0, 2), (0, 3)$. How does the lengths of the gradient vectors change.
   (c) Sketch the gradient for the points $(0, 0), (1, 0), (2, 0), (3, 0)$.
   (d) Let $P = (x_0, y_0)$ any point, describe the gradient vector of f at P in words.
8. Let $f(x, y, z)$ be a function for which we only know that at the point $P = (2, 2, 0)$ the gradient is given by $(-1, 1, 1)$. Sketch the tangent plane to the level curve through the point P.
9. Given a function f and a point x such that $\nabla(f) \neq 0$. What is the direction for which f *decreases* the most?
10. The temperature on a desk is described by $T(x, y) = 50 + x^2y^3 - 3x$. An ant is at the position $(3, 5)$ and wants to go on a walk. It's really fond of the temperature at $(3, 5)$ and therefore wants to change the temperature as little as possible. In which direction should the ant go?