Homework 2 – Solutions

2. p. 37, problem 18.
Solution: Let's first try to find the intersection point (x_0, y_0, z_0). It has to satisfy the following two conditions:

   (a) $(x_0, y_0, z_0) = (-1+t, -2+t, -1+t)$ for some $t \in \mathbb{R}$ since the point lies on the given line.

   (b) The vector connecting $(3, 1, -2)$ to (x_0, y_0, z_0) has to be perpendicular to the direction vector of the line, i.e. it has to be perpendicular to the vector $(1, 1, 1)$. (Note that the line is given by $(-1+t, -2+t, -1+t) = (-1, -2, -1)+t(1, 1, 1)$). So we get the equation

$$(3 - x_0, 1 - y_0, -2 - z_0) \cdot (1, 1, 1) = 0.$$

Let's put these two conditions together, we get

$$\begin{array}{rrcl} & (3 - (-1 + t), 1 - (-2 + t), -2 - (-1 + t)) \cdot (1, 1, 1) & = & 0 \\ \Rightarrow & (4 - t) + (3 - t) + (-1 - t) & = & 0 \\ \Rightarrow & t & = & 2. \end{array}$$

So the intersection point is

$$(x_0, y_0, z_0) = (-1 + 2, -2 + 2, -1 + 2) = (1, 0, 1).$$

The required line is now the line through the given point $(3, 1, -2)$ and $(1, 0, 1)$. A possible equation is

$$(3, 1, -2) + s(2, 1, -3), s \in \mathbb{R}.$$

7. Let $P = (1, 1, 0)$ and $\mathbf{v} = (-1, 1, 0)$ and $\mathbf{w} = (2, 0, 1)$. Furthermore let $Q = (2, 3, -2)$. Determine whether Q lies on the plane.
Solution: Perhaps the easiest way to solve this problem is to first write down the equation of the plane. We compute a normal vector to be

$$\mathbf{n} = (-1, 1, 0) \times (2, 0, 1) = (1, 1, -2).$$

Then a point (x, y, z) lies on the plane if it satisfies the equation

$$(x - 1, y - 1, z - 0) \cdot (1, 1, -2) = 0,$$

where the first vector is the vector connection (x, y, z) to the base point P. Plugging in $(x, y, z) = Q = (2, 3, -2)$ we see that Q does not lie on the plane.