Homework 1 – Solution

2. Consider the line given by the equation $x = 1 + 2t, y = 2 - 3t$. Determine whether the point $(3, 5)$ lies on the line.
   Solution: We have to check whether there exists $t \in \mathbb{R}$ such that

$$\begin{aligned} 3 &= 1 + 2t, \text{ AND} \\ 5 &= 2 - 3t. \end{aligned}$$

   The first equation gives $t = 1$ and the second equation gives $t = -1$. Since t can not be 1 and -1 at the same time, there exists no solution, so the point does not lie on the line.

3. Let $P = (2, 3, -1)$ and $Q = (8, 6, -4)$. Check whether the point $R = (4, 4, -2)$ lies on the line segment $\overline{PQ}$ or not.
   Solution: Now we have to check whether there exists $t \in [0, 1]$ (note the restriction!) such that $R = P + t\mathbf{PQ}$. In coordinates, we have to check whether there exists $t \in [0, 1]$ such that

$$\begin{aligned} 4 &= 2 + 6t \\ 4 &= 3 + 3t \\ -2 &= -1 + (-3)t. \end{aligned}$$

   It's easy to check that the solution is $t = \frac{1}{3}$, which lies in the interval $[0, 1]$ so R lies on the line segment.

6. Let l be the line through $P = (2, 1)$ and $Q = (0, -3)$. Find the point R on the line l such that the vector $\mathbf{0R}$ is orthogonal to the vector $\mathbf{v} = (1, -1)$. More precisely, do the following steps:

   b. Write down the equation of the line l, i.e. write down the points (x, y) on the line in terms of the parameter t.
      Solution: The direction vector for the line is $\mathbf{w} = P - Q = (2, 4)$. We now have $(x, y) = P + t\mathbf{w}$, i.e. the equations are

$$\begin{aligned} x &= 2 + 2t \\ y &= 1 + 4t, \end{aligned}$$

      where t is any real number.

   c. For a point $R = (x, y)$ on the line l write down the vector $\mathbf{OR}$.
      Solution: For $R = (x, y) = (2 + 2t, 1 + 4t)$ we have $mathbfOR = R - O = R - (0, 0) = (2 + 2t, 1 + 4t)$.

d. Determine for which t we have $\mathbf{v} \cdot \mathbf{OR} = 0$.
Solution:
We have

$$\mathbf{v} \cdot \mathbf{OR} = (1, -1) \cdot (2 + 2t, 1 + 4t) = 2 + 2t - 1 - 4t = 1 - 2t.$$

This equals zero when $t = \frac{1}{2}$. So the required point on the line is $R = (2 + 2\frac{1}{2}, 1 + 4\frac{1}{2}) = (3, 3)$.