Homework 14 – this homework will not be collected

1. Compute $\int_C \mathbf{F}(x,y)\cdot d\mathbf{s}$ for $\mathbf{F}(x,y) = (x-y, y-x)$ and C the square bounded by $x=0, x=1, y=0, y=1$.
2. Compute $\int_C \mathbf{F}(x,y)\cdot d\mathbf{s}$ for $\mathbf{F}(x,y) = (\tan^{-1}(\frac{y}{x}), \ln(x^2+y^2))$ and C the boundary of the region defined by the polar coordinate inequalities $1 \le r \le 2$, $0 \le \theta \le \pi$.
3. p. 528, problem 3 (a), (b)
4. p. 528, problem (1), (5)
5. Compute the area of the ellipse given by $\mathbf{c}(t) = (a\cos(t), b\sin(t)), t \in [0, 2\pi]$.
6. p. 547, problems 1, 2
7. p. 547, problem 3. You can make use of the fact, that a parametrization for the surface is given by
$$\Phi(\theta,\phi) = (\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi))$$
and that
$$\begin{aligned}\mathbf{T}_\theta &= (-\sin(\theta)\sin(\phi), \cos(\theta)\sin(\phi), 0)\\ \mathbf{T}_\phi &= (\cos(\theta)\cos(\phi), \sin(\theta)\cos(\phi), -\sin(\phi))\\ \mathbf{T}_\theta\times\mathbf{T}_\phi &= (-\cos(\theta)\sin^2(\phi), -\sin(\theta)\sin^2(\phi), -\sin(\phi)\cos(\phi))\\ &= -\sin(\phi)\,(\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi))\\ &= -\sin(\phi)\,\Phi(\theta,\phi).\end{aligned}$$
Note that the last formula gives an easy way to remember $\mathbf{T}_\theta\times\mathbf{T}_\phi$. You can also use that $T_\theta \times T_\phi$ points inward (this can be seen by considering the last line and noticing that $-\sin(\phi)$ is negative).
8. p. 547, problem 5
9. p. 547, problems 7, 8
10. p. 548, problem 12 (think about it first!)
11. p. 548, problem 15
12. p. 549, problem 17
13. p. 558, problems 2, 3, 4
14. p. 559, problems 7, 13 (a), (b)