2. Let \( F(x, y, z) = (x^2, y - z, 1) \). Let \( S \) be the graph of \( f(x, y) = x^3 - 2xy \) with domain \( D = [0, 1] \times [0, 3] \). Compute \( \int_S F \cdot dS \).

Solution: We first need a parametrization of \( S \). We can describe \( S \), since it's a graph, in terms of \( x, y \). We get
\[
\begin{align*}
x &= x \\
y &= y \\
z &= x^3 - 2xy,
\end{align*}
\]
with \( x \in [0, 1] \) and \( y \in [0, 3] \) (from the domain of \( f \)). So we get
\[
\Phi(x, y) = (x, y, x^3 - 2xy).
\]

We compute
\[
\begin{align*}
T_x &= (1, 0, 3x^2 - 2y) \\
T_y &= (0, 1, -2x) \\
T_x \times T_y &= (-3x^2 + 2y, 2x, 1).
\end{align*}
\]

So we get
\[
\int_S F \cdot dS = \int_{x=0}^{x=1} \int_{y=0}^{y=3} F(\Phi(x, y)) \cdot (T_x \times T_y) \, dx \, dy
\]
\[
= \int_{x=0}^{x=1} \int_{y=0}^{y=3} (x^2, y - x^3 + 2xy, 1) \cdot (-3x^2 + 2y, 2x, 1) \, dx \, dy
\]
\[
= \int_{x=0}^{x=1} \int_{y=0}^{y=3} -3x^4 + 2x^2y + 2xy = 2x^4 + 4x^2y + 1 \, dx \, dy.
\]

3. Let \( C \) be the coke can with radius 3 and with top and bottom at \( z = 1 \) and \( z = 0 \). Give it the usual orientation. Let \( F(x, y, z) = (0, 0, z) \). Compute \( \int_C F \cdot d\mathbf{S} \). If you think about what is happening geometrically, then you should be able to solve this problem with barely any computations. Explain carefully what you are doing. (Hint: sketch \( C \) and \( F \)).

Solution: \( F \) is parallel to the side of the cylinder, so nothing flows in or out. Now note that at the bottom \( z = 0 \), so the vector field is zero, and the flow is hence zero. At the top we have \( F(x, y, z) = (0, 0, 1) \) since \( z = 1 \). Since the flow is perpendicular to the top we see that the flux is just 1 times the area of the top, so we get \( 1 \cdot 3^2 \pi = 9 \pi \).