Homework 13, due Friday 4/21

1. Let $\mathbf{F}(x, y, z) = (x, y, z)$. Let S be the sphere of radius 2. Compute $\int\int_S \mathbf{F} \cdot d\mathbf{S}$.

2. Let $\mathbf{F}(x, y, z) = (x^2, y - z, 1)$. Let S be the graph of $f(x, y) = x^3 - 2xy$ with domain $D = [0, 1] \times [0, 3]$. Compute $\int\int_S \mathbf{F} \cdot d\mathbf{S}$.

3. Let C be the coke can with radius 3 and with top and bottom at $z = 1$ and $z = 0$. Give it the usual orientation. Let $F(x, y, z) = (0, 0, z)$. Compute $\int\int_C \mathbf{F} d\mathbf{S}$. If you think about what is happening geometrically, then you should be able to solve this problem with barely any computations. Explain carefully what you are doing. (Hint: sketch C and $\mathbf{F}$).

4. p. 497, problem 3

5. Let S be the triangle given by the points $P = (1, 0, 0)$, $Q = (0, 1, 0)$ and $R = (0, 0, 1)$.
   (a) Find a parametrization for S. (the difficult part here is to get the domain of your two variables right).
   (b) Now assume S has the orientation which points away from the origin. Let $\mathbf{F}(x, y, z) = (x, y, 0)$. What sign does the flux integral of $\mathbf{F}$ over the oriented surface S have? (sketch $\mathbf{F}$ and you can answer this question without calculations).
   (c) Now compute flux integral of $\mathbf{F}$ over the oriented surface S.
   (d) Now let's give S the opposite orientation (i.e. the normal vector points towards the origin). What is the flux integral of $\mathbf{F}$ over the oriented surface S.