Homework 10 – Solution

3. Let C be the triangle given by $A = (0,0), B = (0,1), C = (1,1)$. View C as a directed simple curve with counterclockwise orientation. Compute $\int_C \mathbf{F} \cdot d\mathbf{s}$ with $\mathbf{F}(x,y) = (\sin(x)y, -yx)$.
Compute: We break the triangle into three line segments and find a parametrization for each of them. We have one line segment connecting $A = (0,0)$ to $C = (1,1)$. We take

$$\mathbf{c}(t) = (t,t), t \in [0,1],$$

clearly $\mathbf{c}(t)$ connects A to C in the right direction. Now we compute

$$\begin{aligned} \int_{t=0}^{t=1} \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)dt &= \int_{t=0}^{t=1} \mathbf{F}(t,t) \cdot (1,1)dt \\ &= \int_{t=0}^{t=1} \sin(t)t, -t^2) \cdot (1,1)dt \\ &= \int_{t=0}^{t=1} \sin(t)t - t^2 dt. \end{aligned}$$

Now we are down to a one–variable integral.
The other two pieces are similar, here I just give the parametrizations (but note that there are many possible ways to parametrize a curve). For the line segment connecting $C = (1,1)$ to $B = (0,1)$ (recall that we go counter–clockwise) we take

$$\mathbf{c}(t) = (1-t, 1), t \in [0,1],$$

and for the line segment connecting $B = (0,1)$ to $A = (0,0)$ (recall that we go counter–clockwise) we take

$$\mathbf{c}(t) = (0, 1-t), t \in [0,1].$$

Now compute the the three line integrals and add up.

4. We saw in class that a line integral $\int \mathbf{F} \cdot d\mathbf{s}$ changes the sign if we change the orientation of the curve. What happens for a path integral, if one changes the orientation?
The path integral does not change.

1. Let c be a curve from a point P to a point Q. Furthermore let $\mathbf{F}(x,y,z)$ be a vector field, and $f(x,y,z)$ a function. Now assume we walk from P to Q, in the middle we realize that we forgot our wallet, walk back to P and then walk from P to Q. How does this detour affect the line integral of $\mathbf{F}(x,y,z)$, and how does it affect the path integral of $f(x,y,z)$.
The line integral is not affected, when we go back and forth we gain and loose the same amount of energy. On the other hand the path integral changes, since going along a curve in either direction gives the same integral, i.e. the contributions to the integral from going back to the starting point and then forward again do NOT cancel.