Math 212 Spring 2006 **Exam #3**

Name: ______________________________ **Section**: ______________

Instructions: You have **2 hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 6 questions. Except for the first problem (True/False), you must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. The exam is due by **Wednesday, April 19, 4 p.m.** Before turning in the exam, be sure to:

- Staple your exam with this cover sheet on top,
- Pledge your exam,
- Write your name and section number above.

The exam is due by **Wednesday, 4 p.m.** Good luck!

Pledge:

Problem	**Value**	**Score**
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. Read each of the following sentences. Circle **True** if the sentence is always true in all cases, or **False** if the sentence is possibly false.

   Let $f(x, y, z)$ be a scalar function, and let $\mathbf{F}(x, y, z)$ be a vector field. (Assume both f and $\mathbf{F}$ have continuous partial derivatives of all orders.) Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in $\mathbb{R}^3$.

   (a) curl grad $f = \mathbf{0}$.

   True **False**

   (b) div grad $f = 0$.

   True **False**

   (c) div curl $\mathbf{F} = 0$.

   True **False**

   (d) Let C be an oriented curve. The path integral of f along C does not change when the orientation of C is reversed.

   True **False**

   (e) Let C be an oriented curve. The line integral of $\mathbf{F}$ along C does not change when the orientation of C is reversed.

   True **False**

   (f) The expression $\mathbf{u} \cdot \mathbf{v}$ is a vector.

   True **False**

   (g) The expression $\mathbf{u} \times \mathbf{v}$ is a vector.

   True **False**

   (h) The expression $(\mathbf{v} \cdot \mathbf{w})\mathbf{u}$ is a vector.

   True **False**

   (i) Let S be an oriented surface. The quantity $\iint_S \mathbf{F} \cdot d\mathbf{S}$ is a vector.

   True **False**

   (j) Let S be an oriented surface. The quantity $\iint_S f\, dS$ is a vector.

   True **False**

2. Let $\mathbf{F}(x, y, z) = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right)$.

   (a) Show curl $\mathbf{F} = (0, 0, 0)$.

   (b) Let C be the unit circle in the xy–plane, oriented **clockwise**. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$.

   (c) Using your answer from (b), explain why $\mathbf{F}$ is not a gradient field, even though curl $\mathbf{F} = (0, 0, 0)$.

3. The surface S is parameterized by $\mathbf{\Phi}(u, v) = (e^u - 2, 2v + 3, 5 + u^2 + v^2)$ with $u, v \in \mathbb{R}$.

   (a) Determine the equation of the tangent plane to $(-1, 5, 6) \in S$.

   (b) Find all points on S for which the tangent plane is parallel to the xy–plane.

4. Let $f(x, y) = \frac{1}{3}x^3 + y\sqrt{2} + 3$, and let D be the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$. Let S be the surface given by the graph of $f(x, y)$ over D.

   (a) Find a parametrization of S.

   (b) Compute $\iint_S 4x^2 \, dS$.

5. Consider the solid hemisphere formed by taking the portion of the unit ball with $y \geq 0$. Let S be the **surface** of this region (so that S is a hemisphere, together with a flat 'base' in the xz–plane). Find the flux of the vector field $\mathbf{V}(x, y, z) = -z\mathbf{i} + \mathbf{j} + x\mathbf{k}$ out of the surface S.

   You may find the following identity useful: $\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$.

6. Let $\mathbf{c}(t) = (1, -t^2, \cos t)$, $0 \leq t \leq \pi$. Evaluate

$$\int_{\mathbf{c}} \sin z \, dx - y^2 \, dy + 3xz \, dz.$$