

**Instructions:** You have **2 hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 6 questions. You must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. The exam is due by **Wednesday, 4 p.m.** Good luck!

1. Find and classify all the critical points of

$$f(x, y) = \frac{1}{2}x^2 - xy + \frac{1}{3}y^3.$$

2. Let  $g(x, y) = 2e^{-x} \cos y$ .

(a) Find the quadratic Taylor polynomial for  $g(x, y)$  around the point  $(0, 0)$ .

(b) Use your answer in part (a) to estimate  $2e^{-0.2} \cos 0.4$ .

3. A tank is in the shape of a half-cylinder of radius 2 and height 3. It is situated in  $\mathbb{R}^3$ , given by the inequalities  $\sqrt{x^2 + y^2} \leq 2$ ,  $y \geq 0$ , and  $0 \leq z \leq 3$ . The temperature at the point  $(x, y, z)$  is given by

$$T(x, y, z) = 2yz^2\sqrt{x^2 + y^2} \text{ }^\circ\text{C}.$$

Find the average temperature in the tank.

4. Let  $T$  be the triangle with vertices  $(0, 0)$ ,  $(1, 1)$  and  $(0, 1)$  and let  $f(x, y) = x \sin(y^3)$ .

(a) Find the correct limits of integration to **set up**  $\iint_T f(x, y) dA$  as a double integral

$$\iint f(x, y) dx dy.$$

(b) Find the correct limits of integration to **set up**  $\iint_T f(x, y) dA$  as a double integral

$$\iint f(x, y) dy dx.$$

(c) Compute  $\iint_T f(x, y) dA$ .

5. Find the maximum and minimum values obtained by  $f(x, y) = x + y^2$  on the ellipse  $x^2 + 3y^2 \leq 9$ .

6. The region  $S$  is cut from a solid ball of radius 1 centered at the origin.  $S$  is the region cut by the inequalities  $z \geq 0$  and  $y \geq x$ . ( $S$  is one-quarter of the entire ball, and contains the point  $(0, 1, 0)$ .)

The mass density of  $S$  at a point  $(x, y, z)$  is given by the function  $\delta(x, y, z) = 30z^2 \text{ kg/m}^3$ .

(a) Find the total mass of  $S$ .

(b) Find the average mass density of  $S$ .