Math 212 Spring 2006
Exam #1

Instructions: You have **90 minutes** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 6 questions. You must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. The exam is due by **Wednesday, 4 p.m.** Good luck!

1. Let $P = (2, 3, 1), Q = (2, 2, 2), R = (3, 3, -1)$.

   (a) Find the equation of the plane through P, Q and R.

   (b) Let l be the line given by $x = 2 - t, y = 3t, z = 1 + 2t$. Find the intersection point of the plane and the line.

2. Let P be the plane given by the equation $x_1 - 2x_2 + 3x_3 + 4 = 0$ and let Q be the plane given by all points of the form

$$(2, 4, 1) + \lambda(3, 3, 1) + \mu(2, -1, 1), \lambda, \mu \in \mathbb{R}.$$

   Determine whether P and Q are parallel or not.

   Note: Two planes are called parallel if they do not intersect, an equivalent condition is that two planes are parallel if their normal vectors are parallel.

3. Consider the two functions $f : \mathbb{R} \to \mathbb{R}^3, f(t) = (\cos(t), \sin(t), t),\ g : \mathbb{R}^3 \to \mathbb{R}, g(x, y, z) = x^2 + y^2 + z^2$.

   (a) Use the chain rule to compute the derivative of $g \circ f$ at the point $t = \pi/4$.

   (b) View the function f as describing a path in 3-space. Write an equation for the tangent line to this path at the point $(0, 1, \pi/2)$.

4. An astronaut is floating in the middle of a nebula in outer space. The gas in this nebula is very hot, and she must decrease the temperature she experiences as quickly as possible. In a rectangular coordinate system centered on her, the temperature of the gas (in degrees Centigrade) is described by the equation

$$T(x, y, z) = -2x + \sin(x^2)y^2 + 2z + 78.$$

   (a) Which direction should the astronaut go? (Note that the astronaut is located at $(0, 0, 0)$).

   (b) Would traveling in the direction of the vector $\mathbf{v} = (-1, -17, 1)$ increase or decrease the temperature she experiences?

5. Consider the graph of the function

$$f(x, y) = x^3(y^2 - 1) + (x - y)^3.$$

   (a) Find the equation of the tangent plane to the graph at $P = (1, 2)$.

   (b) Find a unit vector which is normal to the graph at P.

6. (a) Consider $h(x, y) = x^y$. Find the partial derivatives $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$.

   (b) Use (a) and the Chain Rule to find

$$\frac{d}{dt}\left(f(t)^{g(t)}\right).$$