These are due in class next Thursday 9/18. You may work together. Then write them up on your own.

(1) You may use things proved or quoted as theorems in class: Suppose \( X \) and \( Y \) are submanifolds of a smooth manifold \( Z \) and \( \dim X + \dim Y < \dim Z \). Then the map \( i : X \hookrightarrow Z \) can be slightly altered so that its image is disjoint from \( Y \). On the other hand if \( \dim X + \dim Y = \dim Z \), what can you say?

(2) The Whitney (or direct) sum of vector bundles \( E_0 \) and \( E_1 \) over \( B \) is a vector bundle over \( B \) whose fiber over \( x \) is \( (E_0)_x \oplus (E_1)_x \). It is denoted \( E_0 \oplus E_1 \). (This is not sufficient as a definition but is adequate for this question). Recall that if \( M \) is a submanifold of \( N \) (where \( N \) has a Riemannian metric) then \( N(M \hookrightarrow N) \) is defined as a subbundle of \( T_M(N) \); and \( T(M) \) is naturally isomorphic to a subbundle of \( T_M(N) \). Show that

\[
T_M(N) \cong T(M) \oplus N(M \hookrightarrow N).
\]

Let \( \epsilon^m \) denote any trivial bundle over \( B \) with fiber dimension \( m \). A bundle \( E \) over \( B \) is called **stably trivial** if \( E \oplus \epsilon^k \cong \epsilon^m \) for some \( m \) and \( k \) (obviously \( \text{rank}(E) + k = m \)). Show that the tangent bundle of \( S^n \) is stably trivial as are the tangent bundles of all the orientable surfaces. (Hint: embed). A bundle \( E \) is said to have an inverse bundle, \( -E \), if \( E \oplus -E \cong \epsilon^m \) for some \( m \). Prove that the tangent bundle of any compact manifold has an inverse (same hint).